

## □ 前言

- 为什么需要“大规模计算” [HPC, DL, Business platform system, Cloud已经合流]
  - 导入 – 科学计算(天气预报), DL, 互联网平台(Google, Amazon, Alibaba, MeiTuan, ...)

## □ 基础篇

- 并发程序的样子 – Divide & Conquer, Model & Challenges, PCAM, Data/Task, ...
  - 天气预报的计算
- 运行环境
  - 硬件 – 自己梳理的3个方案 – Shared/Unshared Memory, Hybrid
  - 系统软件 – 协议栈, Modern OS, Distributed Job Scheduler, GTM等

## □ 算法级篇

- OpenMP, MPI, CUDA (DL的实现), Big Data 中的MR/Spark等 (只涉及在Big Data SDK之上的编程; 大数据本身的介绍放到后一部分)

## □ 系统级篇 – 互联网平台的实现

- “秒杀”的技术架构
- 计算广告
- 系统架构 (HTAP等)
  - Flink, ClickHouse, MaxCompute, ELK ...

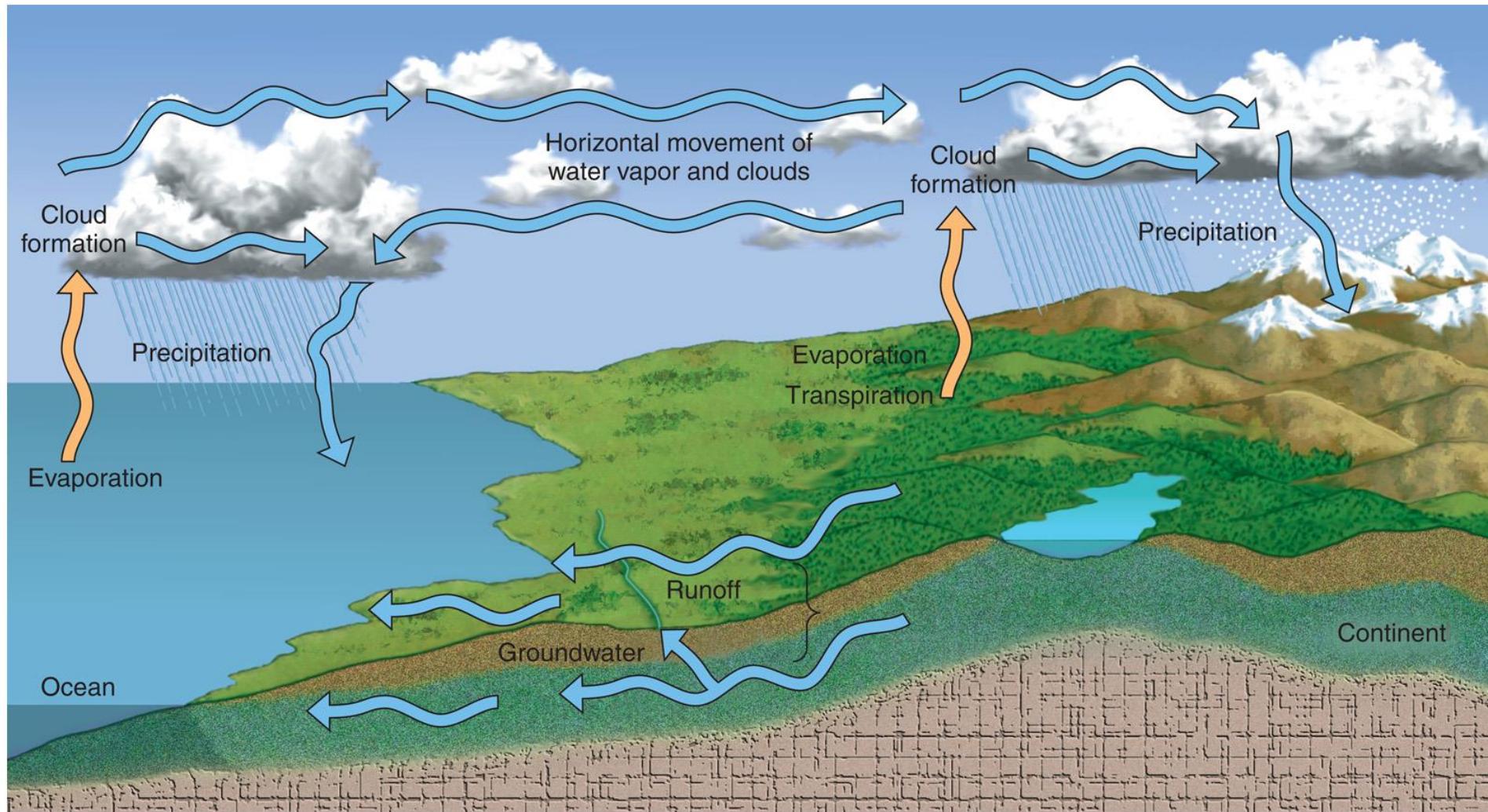
# Chapter 2: HPC with an example

## □ Faster for larger data

- Many problems/applications need HPComputers
  - Weather / Climate, Cryptography, Nuclear Weapons Design, Scientific Simulation, Petroleum Exploration, Aerospace Design, Automotive Design, Pharmaceutical Design, Data Mining, Data Assimilation
- Heat dynamics as an example to understand Numeric Computing/Scientific Computing
  - Like Finite Element Analysis (有限元分析)
- Other examples



# The Hydrologic Cycle



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## □ The Father of Modern Meteorology



## □ Vilhelm Bjerknes' Vision

- 1901 – Wanted to incorporate physics into weather forecasting
  - Start with complete set of initial conditions (3-D)
  - Solve equations using graphical methods
  - Initial state not sufficient for good forecasts
  - Did not use continuity equation to derive the initial vertical wind component (no direct measurements available)



*V. Bjerknes*

Source: Historical Essays on Meteorology 1919-1995, AMS

# No calculus solution!

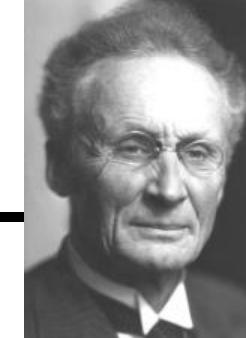
## □ Atmosphere dynamics with considering many parameters:

- 3-D: Temperature, Humidity, Wind speed and Wind direction, Atmospheric Pressure,
- Later even Dew Point, Relative Humidity ...

$$\begin{cases} \frac{du}{dt} - \frac{uv \tan \varphi}{r} + \frac{uw}{r} = -\frac{1}{\rho r \cos \varphi} \frac{\partial p}{\partial \lambda} + fv - \tilde{f}w + F_\lambda \\ \frac{dv}{dt} + \frac{u^2 \tan \varphi}{r} + \frac{vw}{r} = -\frac{1}{\rho r} \frac{\partial p}{\partial \varphi} - fu + F_\varphi \\ \frac{dw}{dt} - \frac{u^2 + v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} - g + \tilde{f}u + F_r \\ \frac{d\rho}{dt} + \rho \left[ \frac{1}{r \cos \varphi} \frac{\partial u}{\partial \lambda} + \frac{1}{r \cos \varphi} \frac{\partial (v \cos \varphi)}{\partial \varphi} + \frac{\partial (wr^2)}{r^2 \partial r} \right] = 0 \\ \frac{dT}{dt} - \frac{RT}{C_p p} \frac{dp}{dt} = \frac{\dot{Q}}{C_p} \\ p = \rho RT \end{cases}$$

## COSMO: the model Meteo Swiss uses

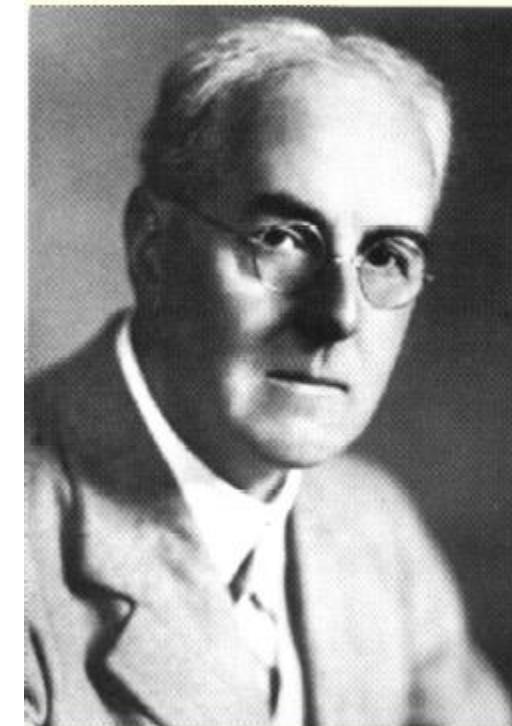
$$\begin{aligned}
 \text{velocities} \quad & \begin{cases} \frac{\partial u}{\partial t} = - \left\{ \frac{1}{a \cos \varphi} \frac{\partial E_h}{\partial \lambda} - v V_s \right\} - \zeta \frac{\partial u}{\partial \zeta} - \frac{1}{\rho a \cos \varphi} \left( \frac{\partial p'}{\partial \lambda} - \frac{1}{\sqrt{\gamma}} \frac{\partial p_0}{\partial \lambda} \frac{\partial p'}{\partial \zeta} \right) + M_u \\ \frac{\partial v}{\partial t} = - \left\{ \frac{1}{a} \frac{\partial E_h}{\partial \varphi} + u V_s \right\} - \zeta \frac{\partial v}{\partial \zeta} - \frac{1}{\rho a} \left( \frac{\partial p'}{\partial \varphi} - \frac{1}{\sqrt{\gamma}} \frac{\partial p_0}{\partial \varphi} \frac{\partial p'}{\partial \zeta} \right) + M_v \\ \frac{\partial w}{\partial t} = - \left\{ \frac{1}{a \cos \varphi} \left( u \frac{\partial w}{\partial \lambda} + v \cos \varphi \frac{\partial w}{\partial \varphi} \right) \right\} - \zeta \frac{\partial w}{\partial \zeta} + \frac{g}{\sqrt{\gamma}} \frac{\rho_0}{\rho} \frac{\partial p'}{\partial \zeta} + M_w + g \frac{\rho_0}{\rho} \left\{ \frac{(T - T_0)}{T} - \frac{T_0 p'}{T p_0} + \left( \frac{R_e}{R_d} - 1 \right) q'' - q^l - q^r \right\} \end{cases} \\
 \text{pressure} \quad & \frac{\partial p'}{\partial t} = - \left\{ \frac{1}{a \cos \varphi} \left( u \frac{\partial p'}{\partial \lambda} + v \cos \varphi \frac{\partial p'}{\partial \varphi} \right) \right\} - \zeta \frac{\partial p'}{\partial \zeta} + g \rho_0 w - \frac{c_{pd}}{c_{vd}} p D \\
 \text{temperature} \quad & \frac{\partial T}{\partial t} = - \left\{ \frac{1}{a \cos \varphi} \left( u \frac{\partial T}{\partial \lambda} + v \cos \varphi \frac{\partial T}{\partial \varphi} \right) \right\} - \zeta \frac{\partial T}{\partial \zeta} - \frac{1}{\rho c_{vd}} p D + Q_T \\
 \text{water} \quad & \begin{cases} \frac{\partial q''}{\partial t} = - \left\{ \frac{1}{a \cos \varphi} \left( u \frac{\partial q''}{\partial \lambda} + v \cos \varphi \frac{\partial q''}{\partial \varphi} \right) \right\} - \zeta \frac{\partial q''}{\partial \zeta} - (S^l + S^r) + M_{q''} \\ \frac{\partial q^{l,r}}{\partial t} = - \left\{ \frac{1}{a \cos \varphi} \left( u \frac{\partial q^{l,r}}{\partial \lambda} + v \cos \varphi \frac{\partial q^{l,r}}{\partial \varphi} \right) \right\} - \zeta \frac{\partial q^{l,r}}{\partial \zeta} - \frac{g}{\sqrt{\gamma}} \frac{\rho_0}{\rho} \frac{\partial P_{l,r}}{\partial \zeta} + S^{l,r} + M_{q^{l,r}} \end{cases} \\
 \text{turbulence} \quad & \frac{\partial e_t}{\partial t} = - \left\{ \frac{1}{a \cos \varphi} \left( u \frac{\partial e_t}{\partial \lambda} + v \cos \varphi \frac{\partial e_t}{\partial \varphi} \right) \right\} - \zeta \frac{\partial e_t}{\partial \zeta} + K_m^e \frac{g \rho_0}{\sqrt{\gamma}} \left\{ \left( \frac{\partial u}{\partial \zeta} \right)^2 + \left( \frac{\partial v}{\partial \zeta} \right)^2 \right\} + \frac{g}{\rho \theta_v} F^{d_t} - \frac{\sqrt{2} c_v^{3/2}}{\alpha_M l} + M_{e_t} \end{aligned}$$



Vilhelm Bjerkenes

## □ Lewis F. Richardson

- About same time as Bjerkenes; **WWI** (World War I) ambulance driver
- Used continuity equation to obtain initial vertical velocities, as well as the other “primitive equations”
- Failed due to insufficient initial data
  - **Solved equations by hand!**
  - **Time steps** were too large – would have resulted in computational instability



Source: Historical Essays on Meteorology 1919-1995, AMS

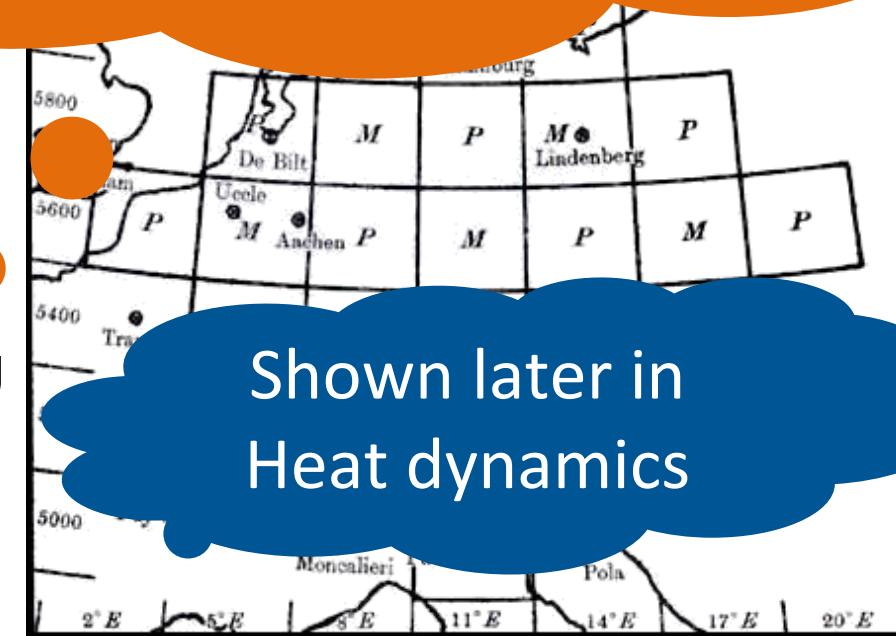
**First numerical forecast  
1922 by Lewis Fry Richardson**

**Took several months, calculating by hand, to produce a 6-hour forecast.**

**It failed...badly!**

**But, it demonstrated the means of producing quantitative forecasts. Its failure has since been shown to be due to the limited understanding of some atmospheric processes at the time.**

Yes, we need numeric weather prediction – by using computers/supercomputers/HPC



L. F. Richardson's computational grid: Pressure is determined in squares marked 'P', momentum in those marked 'M'.

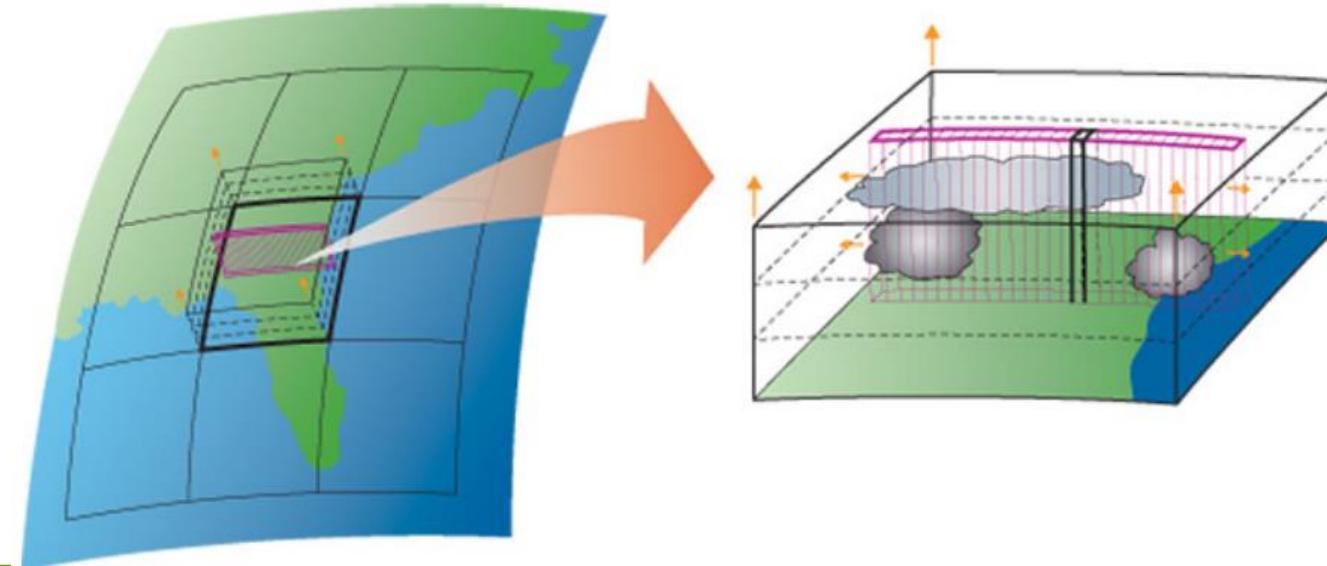
## Richardson's forecast factory (1922)

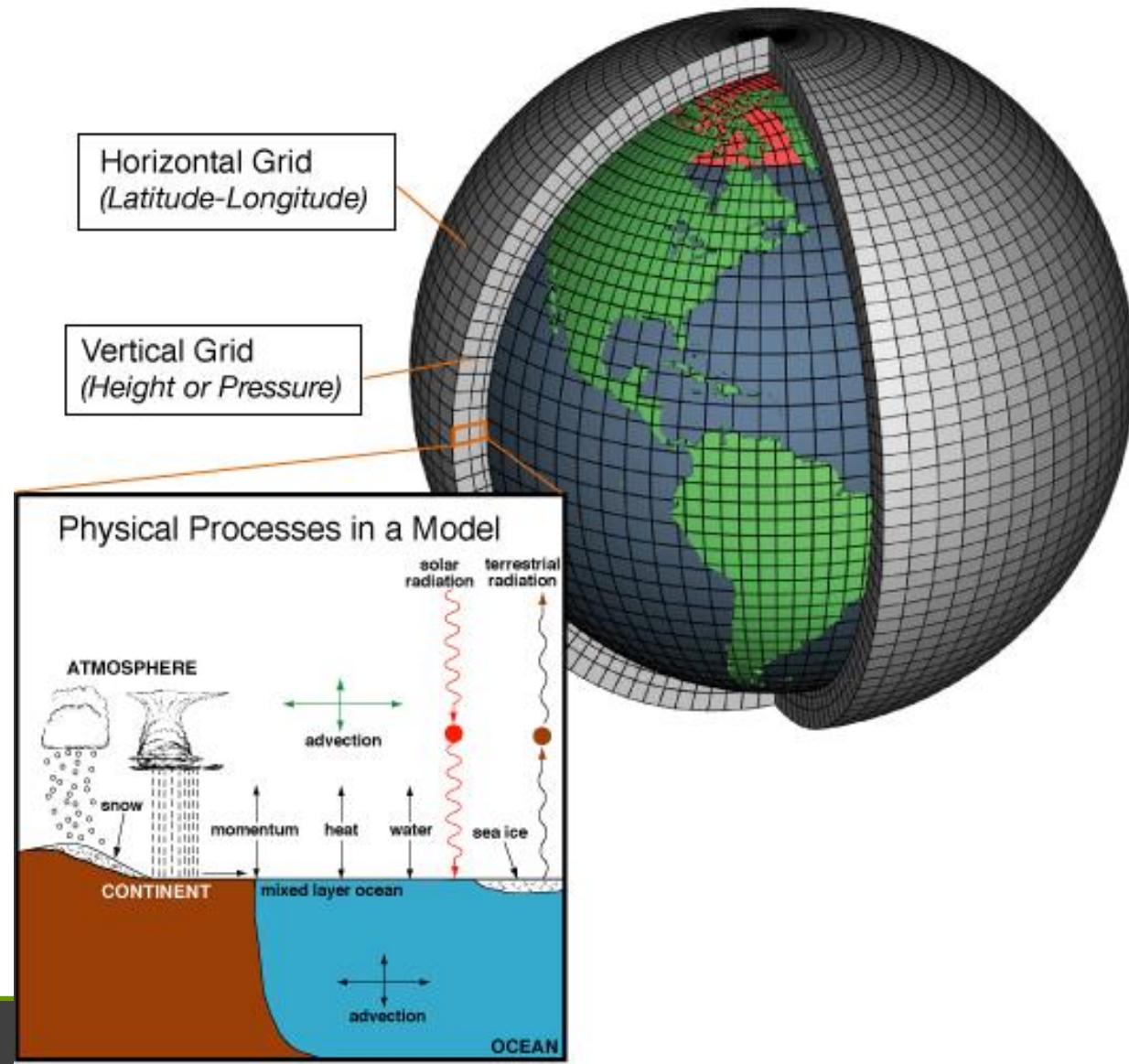
Lewis Fry Richardson: Weather Prediction by Numerical Process



L. Bengtsson & NOAA

# So called Numeric Computing





vorticity 美 [vɔ'tɪsɪtɪ]

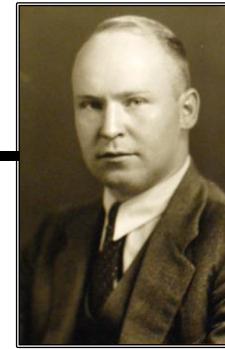
n. 旋涡状态; 涡量; 涡旋; 旋度;

geostrophic 美 [dʒiə'strɒfɪk]

adj. 因地球自转而引起的, 地转风的;

hydrostatic 美 [haɪdrə'stætɪk]

adj. 静水力学的, 流体静力学的;



Rossby in 1933

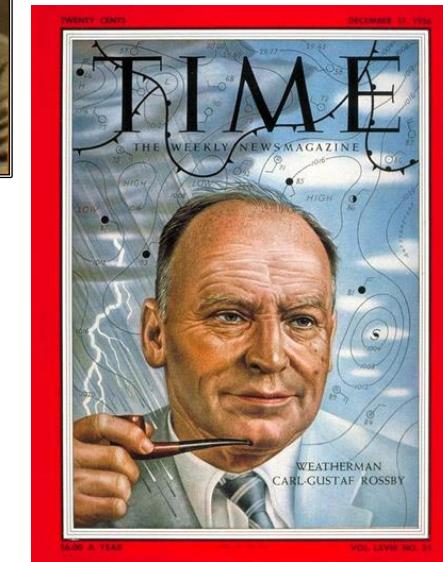
## □ Further Developments

### ■ Carl-Gustaf Rossby (1939)

- Showed that atmospheric longwave motion could be explained by **vorticity** distribution
- Wave movement function of wavelength and speed of large-scale zonal flow (Rossby Waves)

### ■ Jule Charney (1949)

- Developed first **barotropic** model  
barotropic adj. 正压的;
  - ✓ Large-scale motions approximately **geostrophic** and **hydrostatic**; no vertical motions; no vertical wind shear
- Numerical prediction now realizable as soon as computers become powerful enough to run the computations



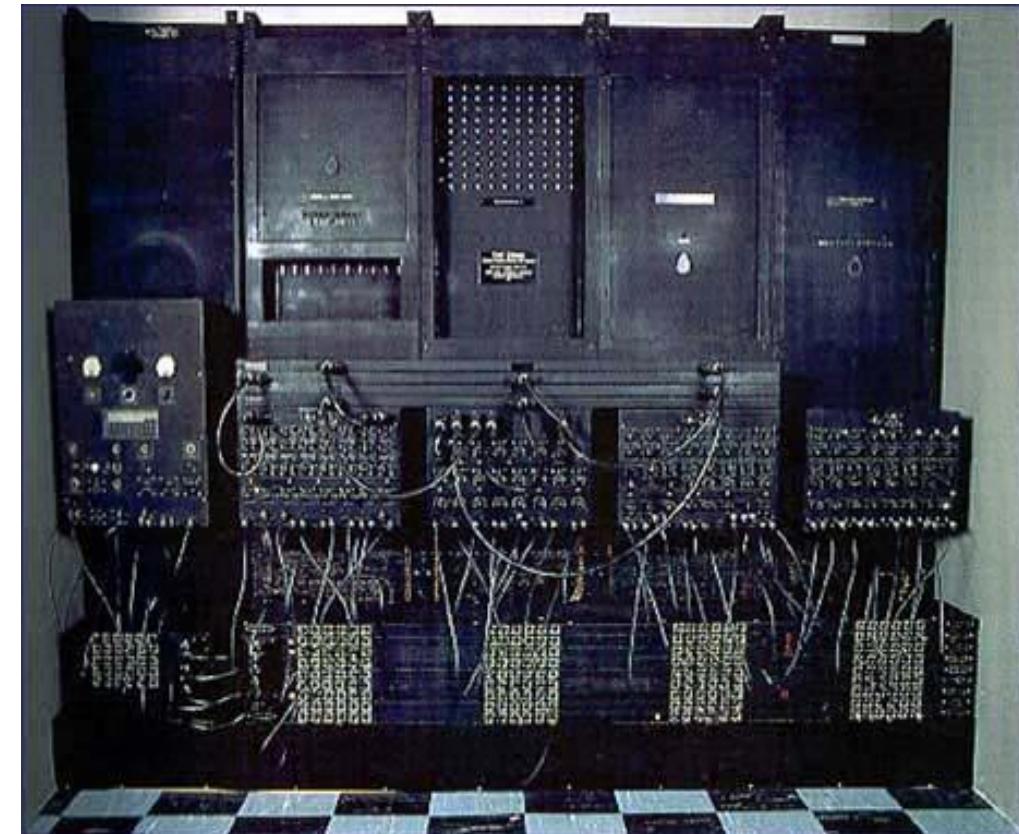
Jule G. Charney

## □ First Numerical Forecast

- Charney barotropic model run on **ENIAC** computer (1950)
  - Produced 500 mb (millibar) height forecast
  - Bad forecast but looked realistic



Jule Charney

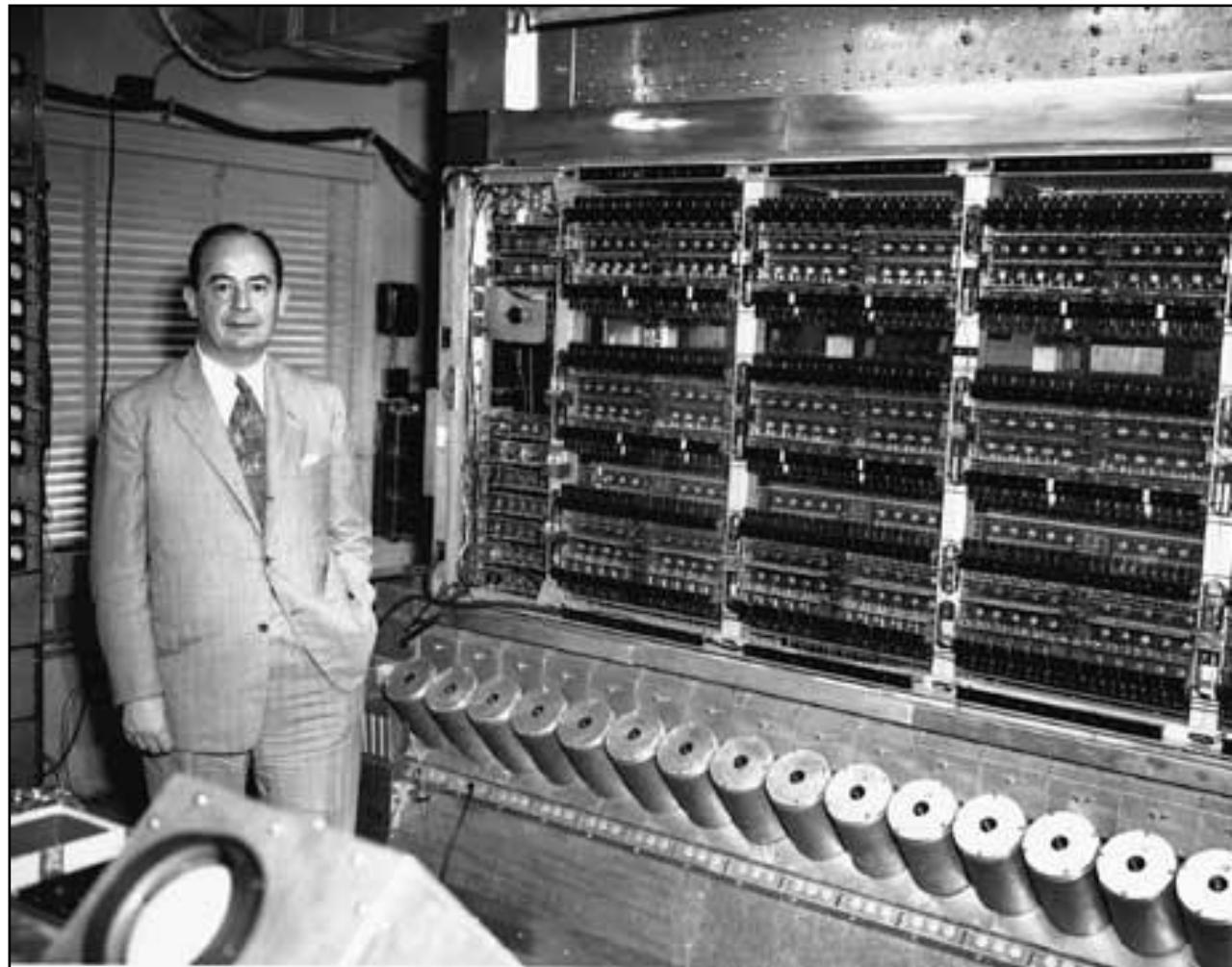
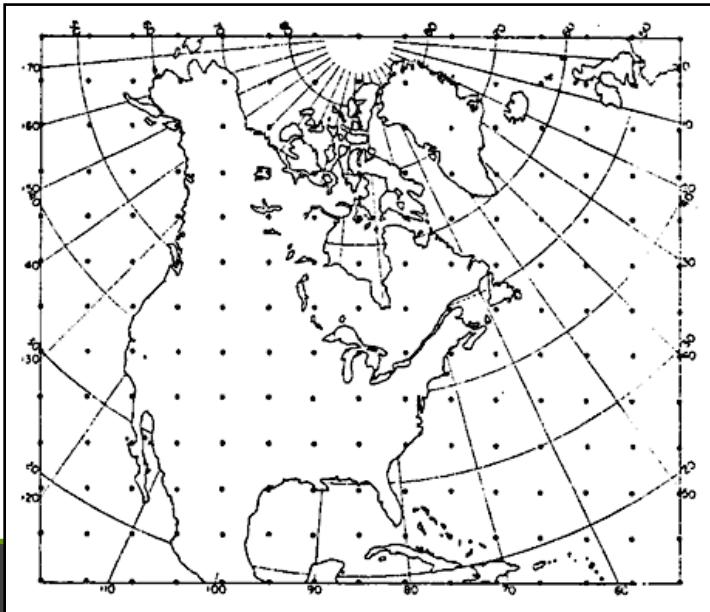


ENIAC Computer

Source: Historical Essays on Meteorology 1919-1995, AMS

First successful forecast: 1950  
by Jule Charney, Fjörtoft, and  
von Neumann, using ENIAC.

A 24-hour forecast took 33  
days to produce, working day  
and night.



# 1950 ENIAC Meteorology Simulations



# Jule Charney was the **NWP** (Numerical weather prediction) super hero



## □ Operational Numerical Weather Prediction

■ May 6, 1955 – First regular and continuing NWP forecasts issued for U.S.

- Early results worse than lab experiments
- Limitations of assumptions made in the models (Quasi-geostrophic approximation)
- Many storms missed; public confidence wanes
- Many early problems due to bad input data from global observation networks
- **Barotropic model** (Charney) worked best for several years
  - ✓ Barotropic processes mostly controlled large-scale daily motions (e.g. long waves), while baroclinic controlled short-bursts of activity (e.g. mid-latitude cyclones)

barotropic adj. 正压的;

## □ Barotropic Model

- Barotropic atmosphere (**constant density/temperature** on **pressure surface**, **no vertical motion**)
- Absolute **vorticity** conserved

$$\frac{d(f + \zeta)}{dt} = 0$$

vorticity D.J.[vɔ:'tɪsəti] n. 旋涡状态

- Somewhat skillful at large-scale wave prediction

$$\text{Wind } \rho \mathbf{v} = -\nabla p + \rho g - 2\Omega \times (\rho \mathbf{v}) + \mathbf{F}$$

$$\text{Pressure } \dot{p} = -(c_{pd}/c_{vd}) p \nabla \cdot \mathbf{v} + (c_{pd}/c_{vd} - 1) Q_h$$

$$\text{Temperature } p c_{pd} T = \dot{p} + Q_s$$

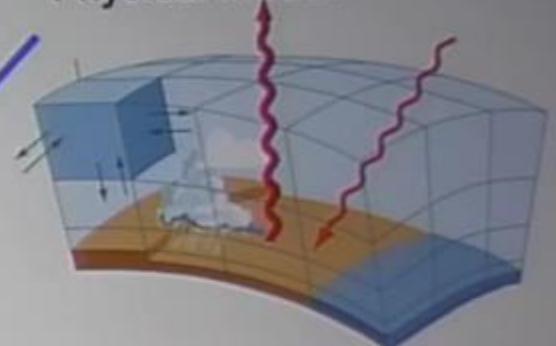
$$\text{Water } \rho \dot{q}^v = -\nabla \cdot \mathbf{F}^v - (J^l + I^l)$$

$$\rho \dot{q}^l = \nabla \cdot (\mathbf{P}^l + \mathbf{F}^l) + I^l$$

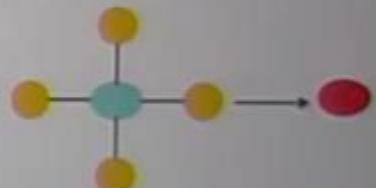
$$\text{Density } \rho = p [R_d (1 + (R_v/R_d - 1) q^v - q^l - q^l) T]^{-1}$$

### Mathematical description

### Physical model



### Algorithmic description



```
    data[i,j,k] = -4.0 * data(i,j,k) +
    data(i+1,j,k) + data(i-1,j,k) +
    data(i,j+1,k) + data(i,j-1,k);
```

### Imperative code



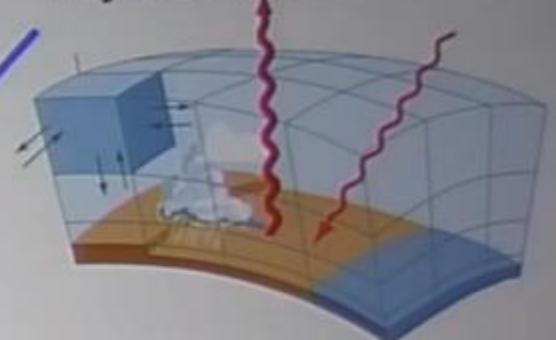
### Compilation

### Computer

Wind  $\rho \dot{\mathbf{v}} = -\nabla p + \rho g - 2\Omega \times (\rho \mathbf{v}) + \mathbf{F}$   
 Pressure  $\dot{p} = -(c_{pd}/c_{vd}) p \nabla \cdot \mathbf{v} + (c_{pd}/c_{vd} - 1) Q_h$   
 Temperature  $\rho c_{pd} \dot{T} = \dot{p} + Q_s$   
 Water  $\rho \dot{q}^v = -\nabla \cdot \mathbf{F}^v - (I^v + I^f)$   
 $\rho \dot{q}^f = \nabla \cdot (\mathbf{P}^f + \mathbf{F}^f) + I^f$   
 Density  $\rho = p [R_d (1 + (R_v/R_d - 1) q^v - q^f) T]^{-1}$

## Mathematical description

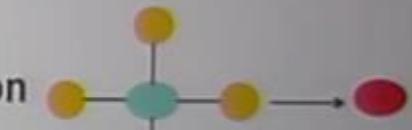
## Physical model



## Domain science &amp; applied mathematics

## Algorithmic description

```
     $\text{data}(i,j,k) = -4.0 * \text{data}(i,j,k) +$ 
     $\text{data}(i+1,j,k) + \text{data}(i-1,j,k) +$ 
     $\text{data}(i,j+1,k) + \text{data}(i,j-1,k);$ 
```



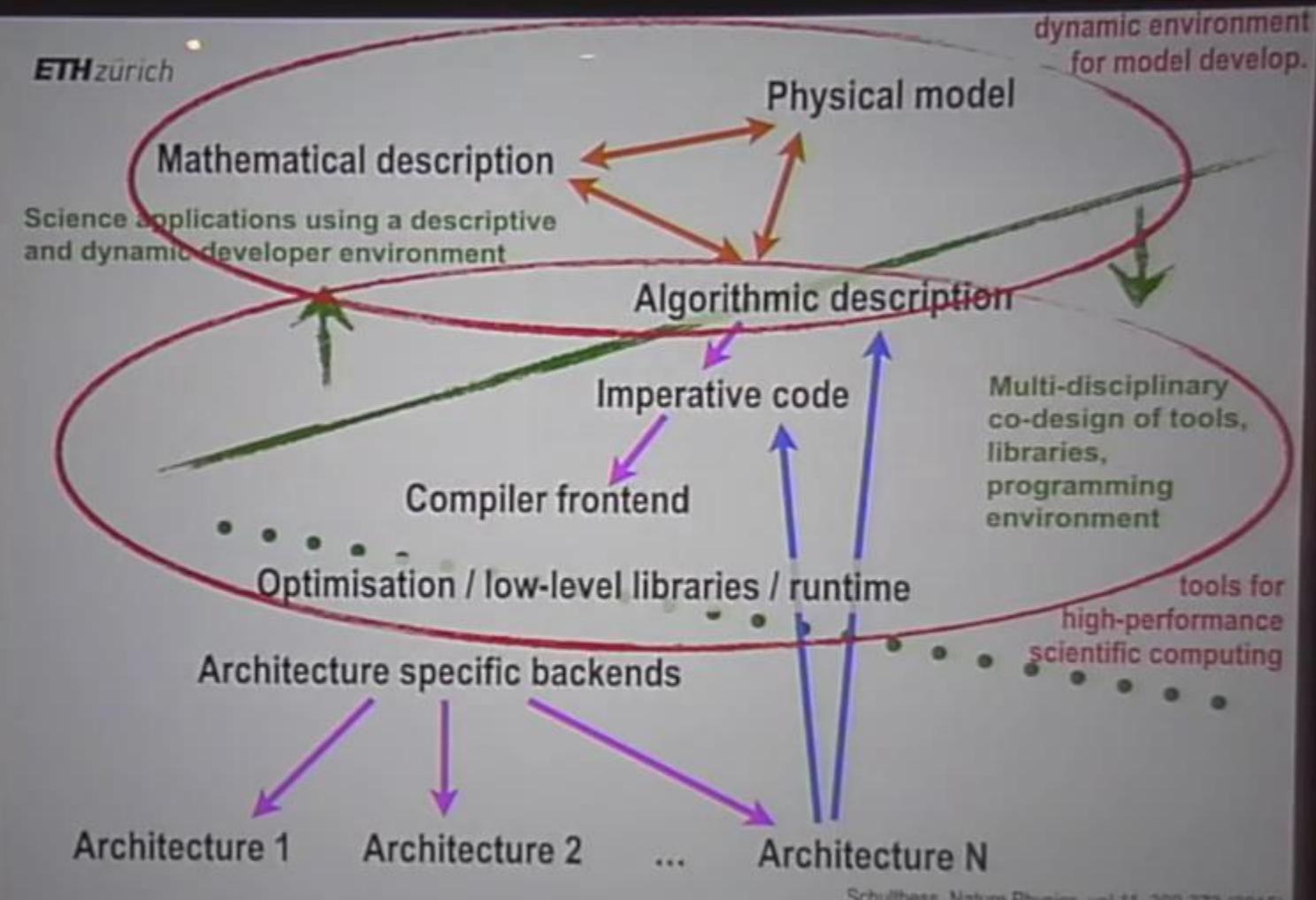
## Imperative code

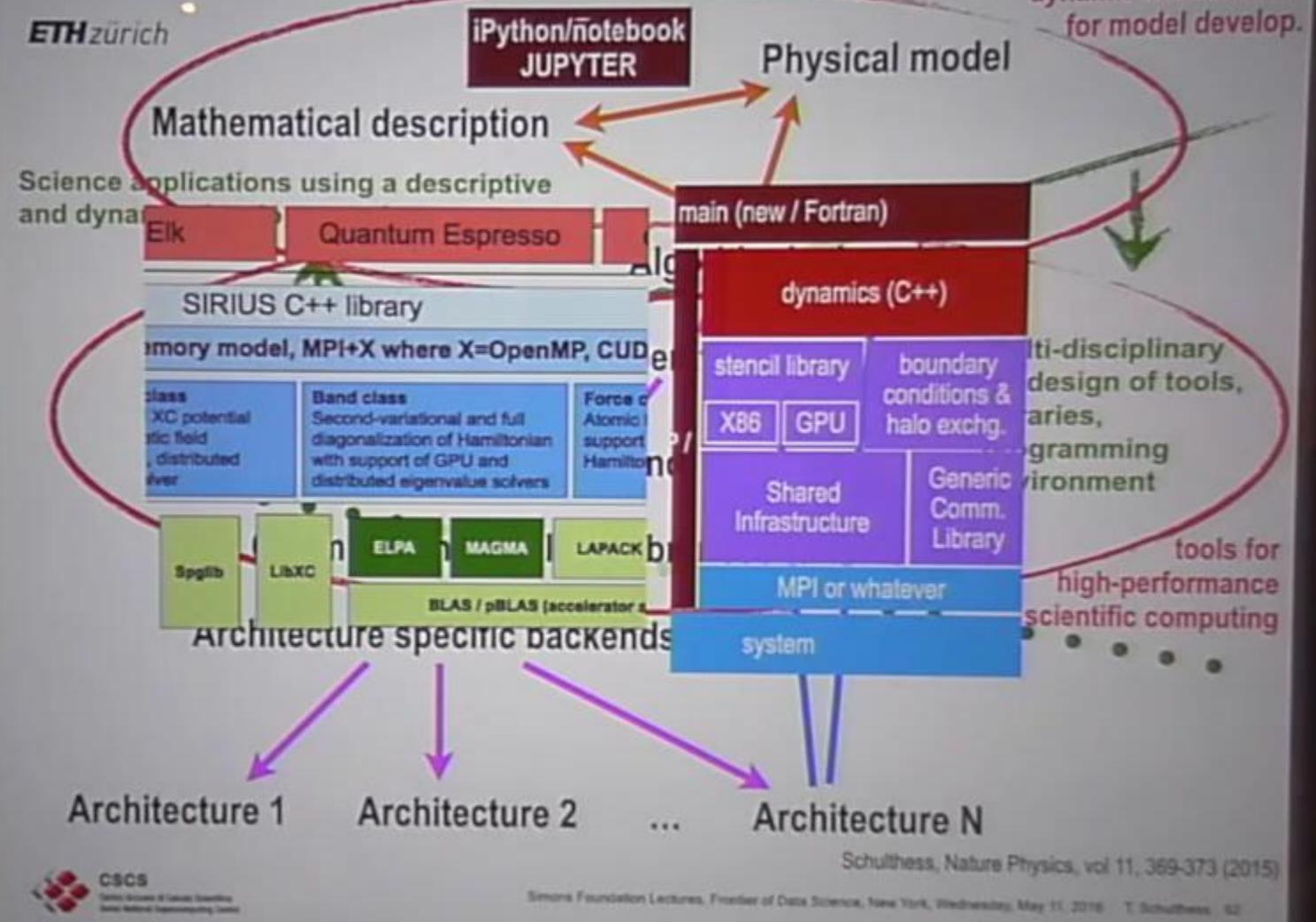


## Computer

## Compilation

## Computer engineering





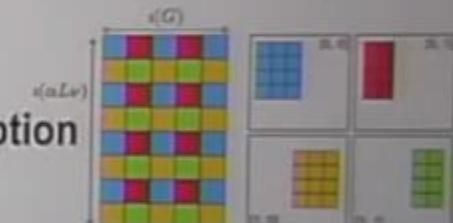
DFT ground state  
 Eigen-value problem  $\hat{H}\Psi_{ik}(\mathbf{r}) = \epsilon_{ik}\Psi_{ik}(\mathbf{r})$   
 Charge density  $\rho_{vv'}(\mathbf{r}) = \sum_{ik} \Psi_{ik}^{*}(\mathbf{r}) \Psi_{ik}^{*}(\mathbf{r})$   
 Effective potential  $v^{eff}(\mathbf{r}) = v^{eff}[\rho_{vv'}(\mathbf{r})](\mathbf{r})$   
 and magnetic field  $\mathbf{B}^{eff}(\mathbf{r}) = \mathbf{B}^{eff}[\rho_{vv'}(\mathbf{r})](\mathbf{r})$

## Mathematical description

## Domain science &amp; applied mathematics



## Algorithmic description



## Imperative code

## Compilation



## Computer

## Computer engineering

Schulthess, Nature Physics, vol 11, 369-373 (2015)

Simons Foundation Lectures, Frontier of Data Science, New York, Wednesday, May 11, 2016 T. Schulthess - 43

$$V(r) = \sum_{\text{bonds}} k_b (b - b_0)^2 + \sum_{\text{angles}} k_\theta (\theta - \theta_0)^2 + \sum_{\text{torsions}} k_\phi (1 + \cos(n\phi - \phi_0)) + \sum_{\text{improper}} k_\psi (\psi - \psi_0)^2 + \sum_{\text{non-bonded}} 4\epsilon_{ij} \left[ \left( \frac{\sigma_{ij}}{r_{ij}} \right)^{12} - \left( \frac{\sigma_{ij}}{r_{ij}} \right)^6 \right] + \sum_{\text{non-bonded}} \frac{q_i q_j}{r_{ij}}$$

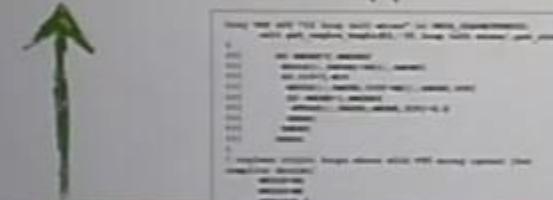


Mathematical description

## Physical model



## Domain science &amp; applied mathematics



## Algorithmic description



## Imperative code



Computer

## Compilation

## Computer engineering

# Types of Numerical Models

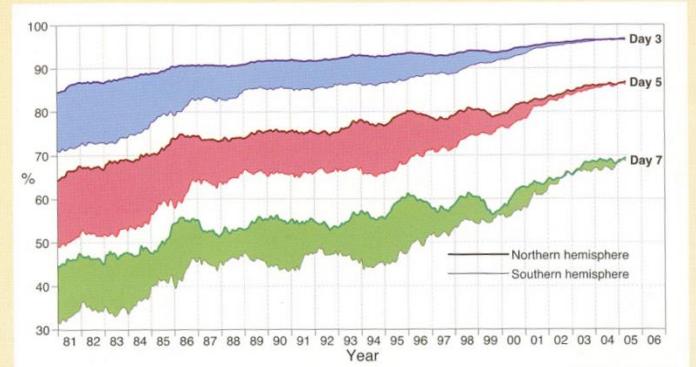
## □ The Final Major Evolution

- Successful NWP using full suite of primitive equations occurred in 1966
  - Original Bjerknes/Richardson vision!
- Advances in computational power and improvements in input data led to acceptable forecasts
- Forecasts improved almost 50% in 10-years (1955-1965) from subjective forecasts

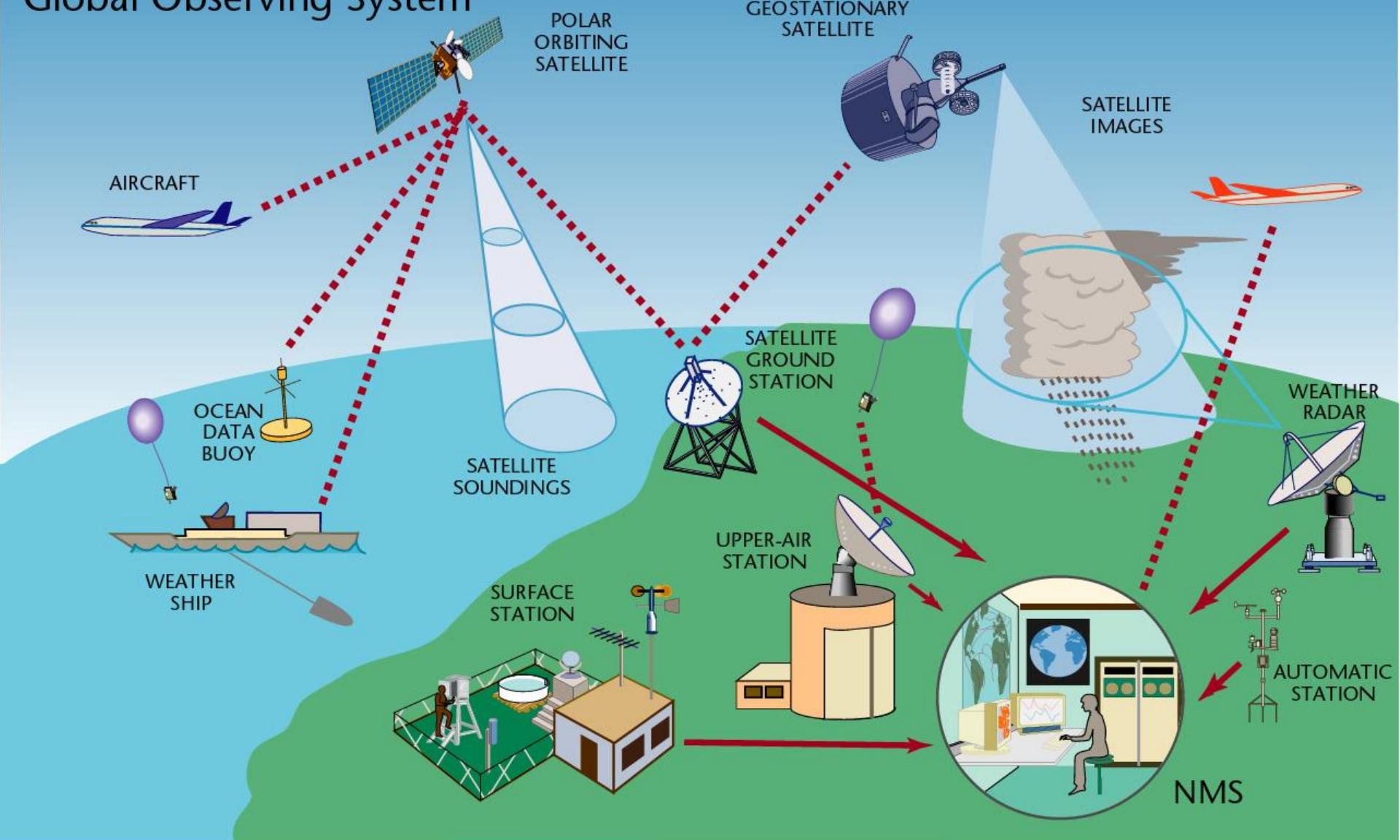
### A tale of progress

One of the greatest scientific achievements and societal benefits provided by Earth science in the past 30 years has been the steady improvement in global weather forecasts. This figure shows the moving monthly average of the correlation (a perfect forecast would be 100%) between observed and forecast weather features at 500 millibars (the approximate midpoint of the atmosphere) for 3-, 5-, and 7-day forecasts made by the European Centre for Medium-Range Weather Forecasts (ECMWF). The accuracy of these forecasts of mid-atmospheric flows—upon which forecasts of hurricanes, floods, droughts, and other major weather features depend—has been increasing steadily since 1980. This dramatic improvement is due to better understanding of the atmosphere-Earth system,

more and better global satellite observations, and improved numerical models that assimilate the many observations. The forecast improvements are especially dramatic across the Southern Hemisphere (bottom line of each colored band), a sign that the benefits of advanced satellite data are available equitably to both North and South. Recent work shows that we are not yet near the end of possible improvements. (Figure courtesy Tony Hollingsworth, ECMWF, and is updated from the figure published by A. J. Simmons and A. Hollingsworth, "Some aspects of the improvement in skill of numerical weather prediction," *Quarterly Journal of the Royal Meteorological Society* 128 (2002), 647–678.)



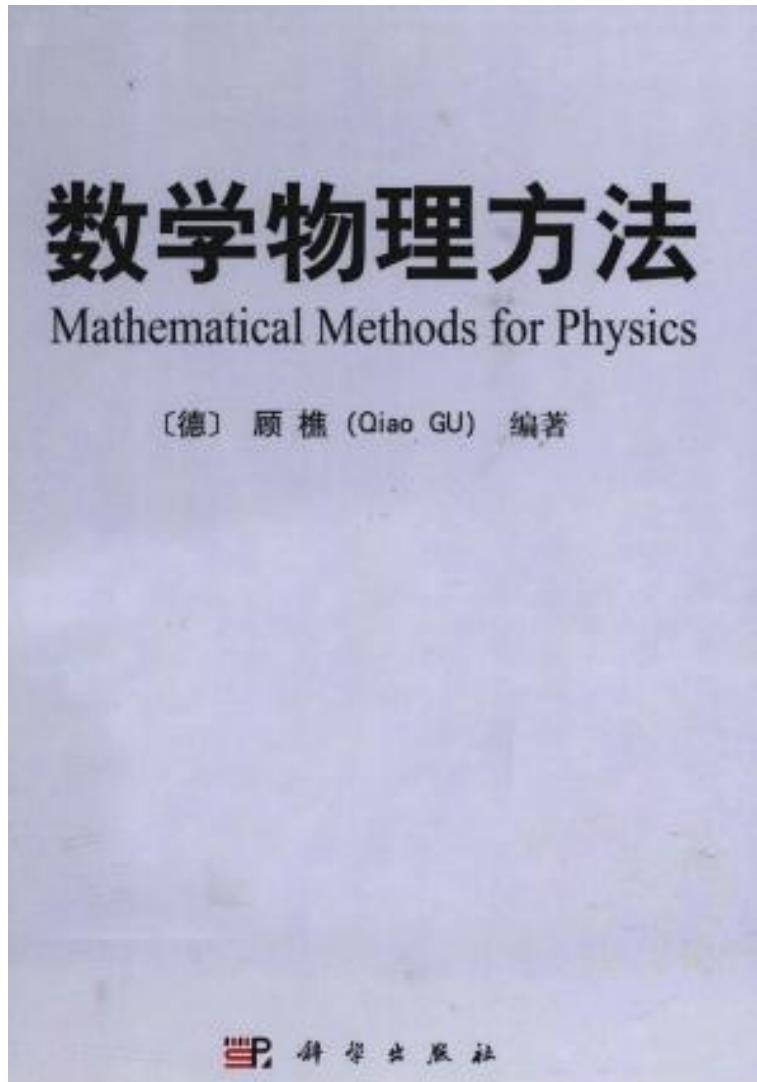
# Global Observing System



# Chapter 2: Overview of HPC

## □ Faster for larger data

- Many problems/applications need HPComputers
  - Weather / Climate, Cryptography, Nuclear Weapons Design, Scientific Simulation, Petroleum Exploration, Aerospace Design, Automotive Design, Pharmaceutical Design, Data Mining, Data Assimilation
- Heat dynamics as an example to understand Numeric Computing/Scientific Computing
  - Such as Finite Element Modeling (FEM: 有限元分析)

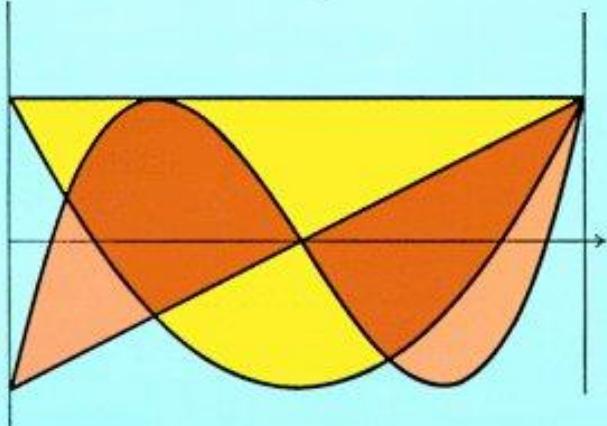


## □ 数学物理方法

## □ 顾樵

■ 由顾樵编著的《数学物理方法》根据作者顾樵20多年来在德国和中国开设数学物理方法讲座内容及相关研究成果提炼而成。其主要内容包括傅里叶级数、傅里叶变换、拉普拉斯变换、数学物理方程的建立、分离变量法、本征函数法、施图姆—刘维尔理论、行波法、积分变换法、格林函数法、贝塞尔函数、勒让德多项式、量子力学薛定谔方程等。本书注重自身理论体系的科学性、严谨性、完整性与实用性，将中国传统教材讲授内容与国外先进教材相结合、教学实践与其他相关课程的需要相结合、抽象的数理概念与直观的物理实例相结合、经典的数理方法与新兴交叉学科的生长点相结合、基础的数理知识与科学前沿中的热点问题相结合。本书既可为教学所用，又可适应科研需要，同时，附有大

# Numerical Methods for Scientists and Engineers



R.W. Hamming

Second Edition

- **Numerical methods for scientists and engineers**
- **Richard Hamming**



- 偏微分方程数值解法
- 第二版
- 陆金甫, 关治



- 数值最优化算法与理论(第二版)
- 李董辉, 童小娇, 万中

# Differential Equation – Basic

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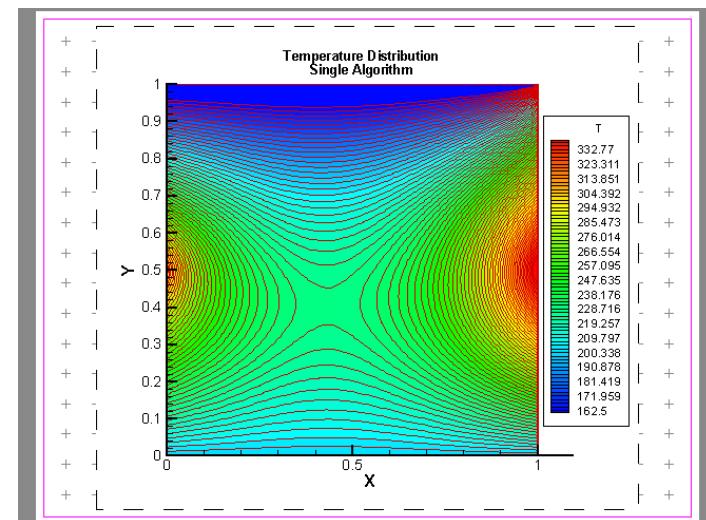
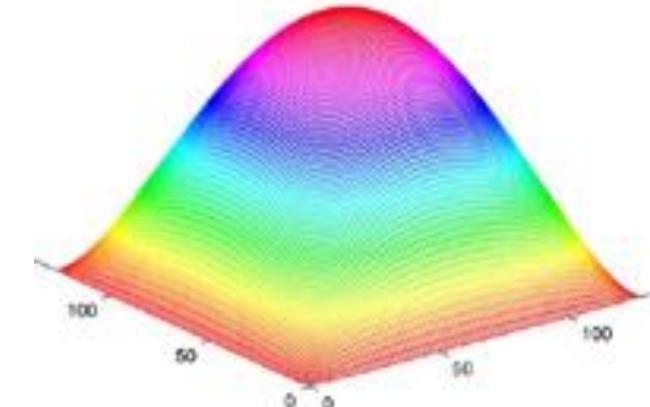
- dimension of unknown:
  - *ordinary differential equation (ODE)* – unknown is a function of one variable, e.g.  $y(t)$
  - *partial differential equation (PDE)* – unknown is a function of multiple variables, e.g.  $u(t,x,y)$
- number of equations:
  - *single* differential equation, e.g.  $y' = y$
  - *system* of differential equations (*coupled*), e.g.  $y_1' = y_2$ ,  $y_2' = -g$
- *order*
  - $n$ th order DE has  $n$ th derivative, and no higher, e.g.  $y'' = -g$

---

- linear & nonlinear:
  - *linear* differential equation: all terms linear in unknown and its derivatives
  - e.g.
    - $x'' + ax' + bx + c = 0$  – linear
    - $x' = t^2 x$  – linear
    - $x'' = 1/x$  – nonlinear

# PDE's in Science & Engineering

- Laplace's Equation:  $\nabla^2 u = u_{xx} + u_{yy} + u_{zz} = 0$ 
  - unknown:  $u(x,y,z)$
  - gravitational / electrostatic potential
- Heat Equation:  $u_t = a^2 \nabla^2 u$ 
  - unknown:  $u(t,x,y,z)$
  - heat conduction
- Wave Equation:  $u_{tt} = a^2 \nabla^2 u$ 
  - unknown:  $u(t,x,y,z)$
  - wave propagation



# Laplace Equation

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$$\frac{\partial^2 u(x, y, z)}{\partial x^2} + \frac{\partial^2 u(x, y, z)}{\partial y^2} + \frac{\partial^2 u(x, y, z)}{\partial z^2} = 0$$

Used to describe the steady state distribution of heat in a body.

Also used to describe the steady state distribution of electrical charge in a body.

# Heat Equation

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$$\frac{\partial u(x, y, z, t)}{\partial t} = \alpha \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

The function  $u(x, y, z, t)$  is used to represent the temperature at time  $t$  in a physical body at a point with coordinates  $(x, y, z)$

$\alpha$  is the thermal diffusivity. It is sufficient to consider the case  $\alpha = 1$ .

# Wave Equation

---

$$\frac{\partial^2 u(x, y, z, t)}{\partial t^2} = c^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

The function  $u(x, y, z, t)$  is used to represent the displacement at time  $t$  of a particle whose position at rest is  $(x, y, z)$ .

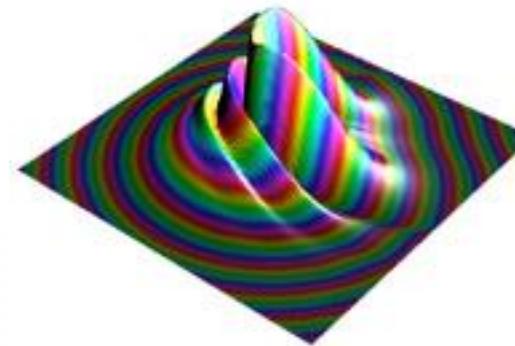
The constant  $c$  represents the propagation speed of the wave.

- Schrödinger Wave Equation

- quantum mechanics
- (electron probability densities)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$

Second derivative with respect to X  
Shrodinger Wave Function  
Position  
Energy  
Potential Energy



- Navier-Stokes Equation

- fluid flow (fluid velocity & pressure)

$$\rho \left[ \frac{\partial V}{\partial t} + (V \cdot \nabla) V \right] = -\nabla P + \rho g + \mu \nabla^2 V$$

change of velocity with time  
Convective term  
Pressure term: Fluid flows in the direction of largest change in pressure  
Body force term: external forces that act on the fluid (such as gravity, electromagnetic, etc.)  
viscosity controlled velocity diffusion term



# Linear or Nonlinear

A PDE is linear if it is linear in the unknown function and its derivatives

Example of linear PDE :

$$2 u_{xx} + 1 u_{xt} + 3 u_{tt} + 4 u_x + \cos(2t) = 0$$

$$2 u_{xx} - 3 u_t + 4 u_x = 0$$

Examples of Nonlinear PDE

$$2 u_{xx} + \underline{(u_{xt})^2} + 3 u_{tt} = 0$$

$$\underline{\sqrt{u_{xx}}} + 2 u_{xt} + 3 u_t = 0$$

$$2 u_{xx} + \underline{2 u_{xt} u_t} + 3 u_t = 0$$

# Convergence and Stability of the Solution

---

## □ Convergence

The solutions converge means that the solution obtained using the finite difference method approaches the true solution as the steps approach zero.

$\Delta x$  and  $\Delta t$

## □ Stability:

An algorithm is stable if the errors at each stage of the computation are not magnified as the computation progresses.

# Laplace Equation

---

**Laplace equation appears in several engineering problems such as:**

- Studying the steady state distribution of heat in a body.
- Studying the steady state distribution of electrical charge in a body.

$$\frac{\partial^2 T(x, y)}{\partial x^2} + \frac{\partial^2 T(x, y)}{\partial y^2} = f(x, y)$$

$T$  : steady state temperature at point  $(x, y)$

$f(x, y)$  : heat source (or heat sink)

# Solution Technique

---

- A grid is used to divide the region of interest.
  - Since the PDE is satisfied at each point in the area, it must be satisfied at each point of the grid.
- A finite difference approximation is obtained at each grid point.

$$\frac{\partial^2 T(x, y)}{\partial x^2} \approx \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2}, \quad \frac{\partial^2 T(x, y)}{\partial y^2} \approx \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y)^2}$$

# Solution Technique

---

$$\frac{\partial^2 T(x, y)}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2},$$

$$\frac{\partial^2 T(x, y)}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y)^2}$$

$$\Rightarrow \frac{\partial^2 T(x, y)}{\partial x^2} + \frac{\partial^2 T(x, y)}{\partial y^2} = 0$$

is approximated by :

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y)^2} = 0$$

# Solution Technique

---

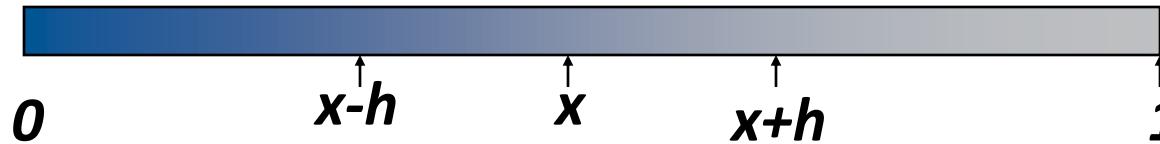
$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y)^2} = 0$$

*(Laplacian Difference Equation)*

Assume :  $\Delta x = \Delta y = h$

$$\Rightarrow T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j} = 0$$

# Example: Deriving the Heat Equation



Consider a simple problem

- A bar of uniform material, insulated except at ends
- Let  $u(x,t)$  be the temperature at position  $x$  at time  $t$
- Heat travels from  $x-h$  to  $x+h$  at rate proportional to:

$$\frac{d u(x,t)}{dt} = C * \frac{(u(x-h,t) - u(x,t))/h - (u(x,t) - u(x+h,t))/h}{h}$$

- As  $h \rightarrow 0$ , we get the heat equation:

$$\frac{d u(x,t)}{dt} = C * \frac{d^2 u(x,t)}{dx^2}$$

# Details of the Explicit Method for Heat

$$\frac{d u(x,t)}{dt} = C * \frac{d^2 u(x,t)}{dx^2}$$

- Discretize time and space using explicit approach (forward Euler) to approximate time derivative:

$$\begin{aligned} (u(x,t+\delta) - u(x,t))/\delta &= C [ (u(x-h,t) - u(x,t))/h - (u(x,t) - u(x+h,t))/h ] / h \\ &= C [u(x-h,t) - 2*u(x,t) + u(x+h,t)]/h^2 \end{aligned}$$

Solve for  $u(x,t+\delta)$  :

$$u(x,t+\delta) = u(x,t) + C * \delta / h^2 * (u(x-h,t) - 2*u(x,t) + u(x+h,t))$$

- Let  $z = C * \delta / h^2$ , simplify:

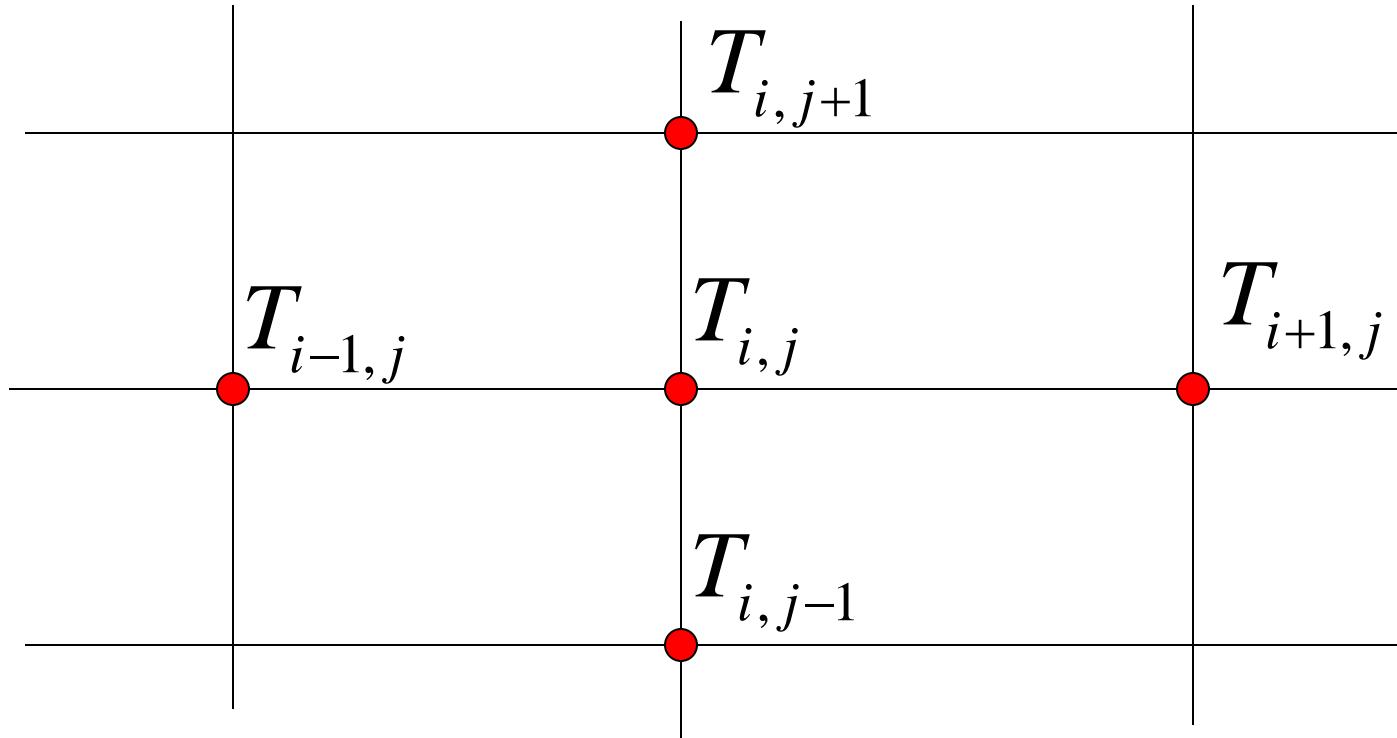
$$u(x,t+\delta) = z * u(x-h,t) + (1-2z) * u(x,t) + z * u(x+h,t)$$

- Change variable  $x$  to  $j * h$ ,  $t$  to  $i * \delta$ , and  $u(x,t)$  to  $u[j,i]$

$$u[j,i+1] = z * u[j-1,i] + (1-2*z) * u[j,i] + z * u[j+1,i]$$

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0$$

# Solution Technique

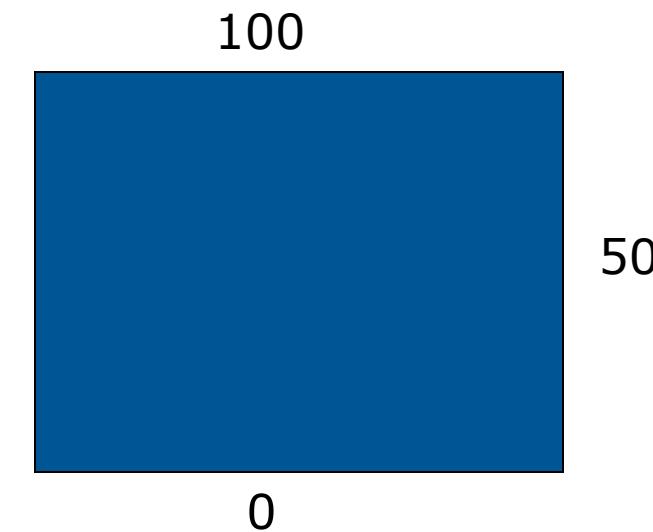


$$T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j} = 0$$

# Example

---

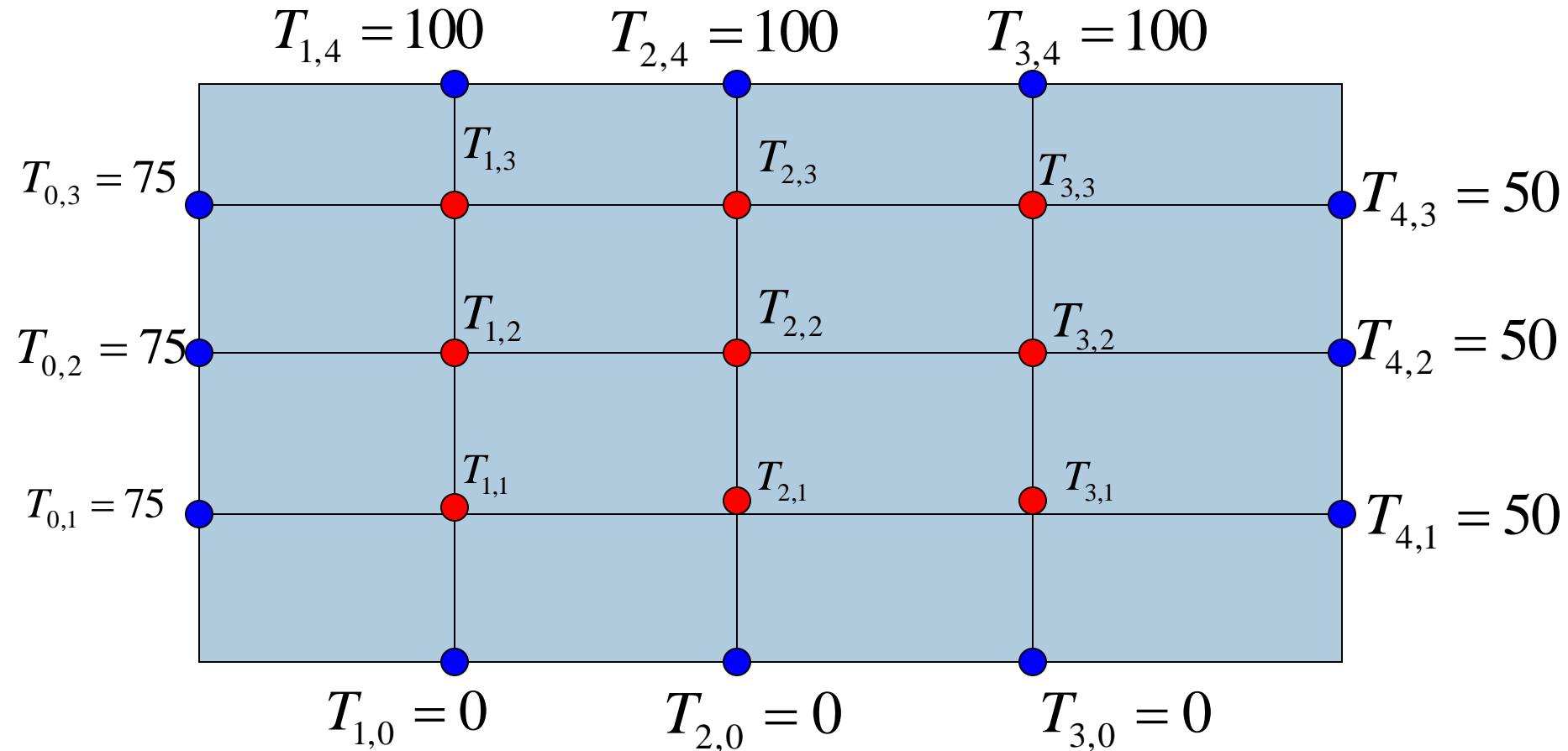
**It is required to determine the steady state temperature at all points of a heated sheet of metal. The edges of the sheet are kept at a constant temperature: 100, 50, 0, and 75 degrees.**



The sheet is divided to 5X5 grids.

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0$$

## Example

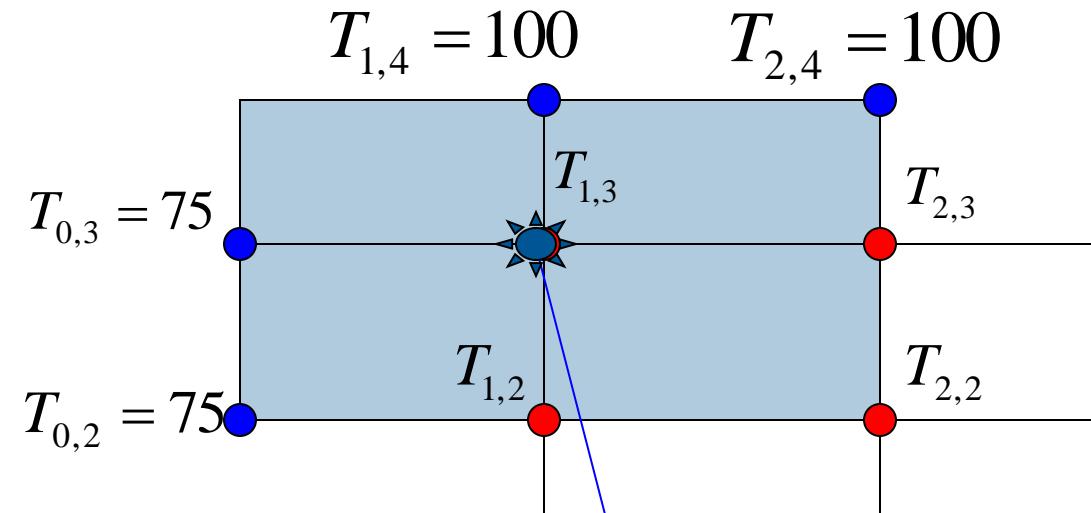


● Known

● To be determined

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0$$

- Known
- To be determined



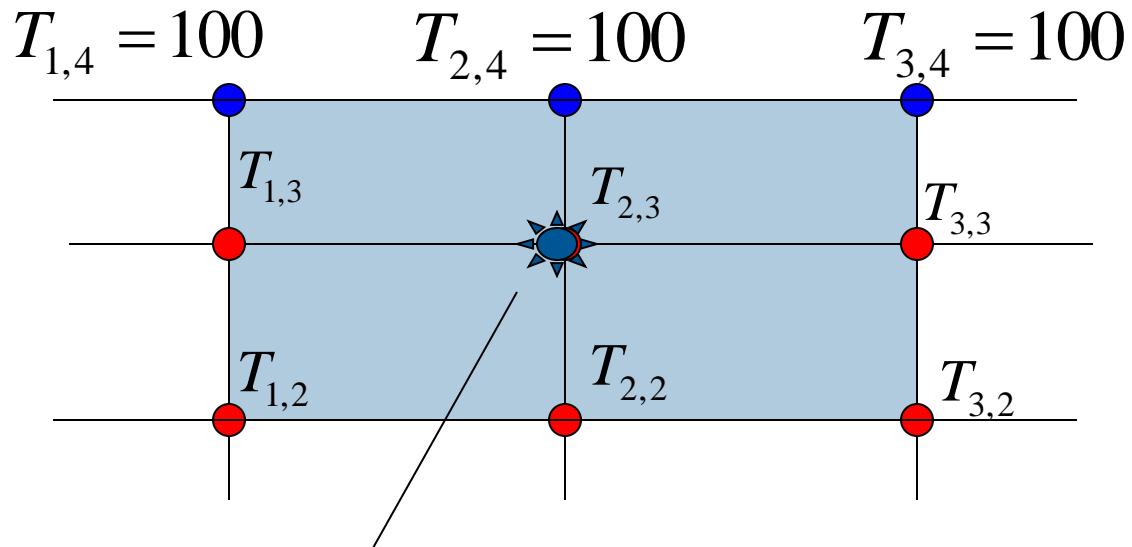
$$T_{0,3} + T_{1,4} + T_{1,2} + T_{2,3} - 4T_{1,3} = 0$$

$$75 + 100 + T_{1,2} + T_{2,3} - 4T_{1,3} = 0$$

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0$$

- Known
- To be determined

## Another Equation

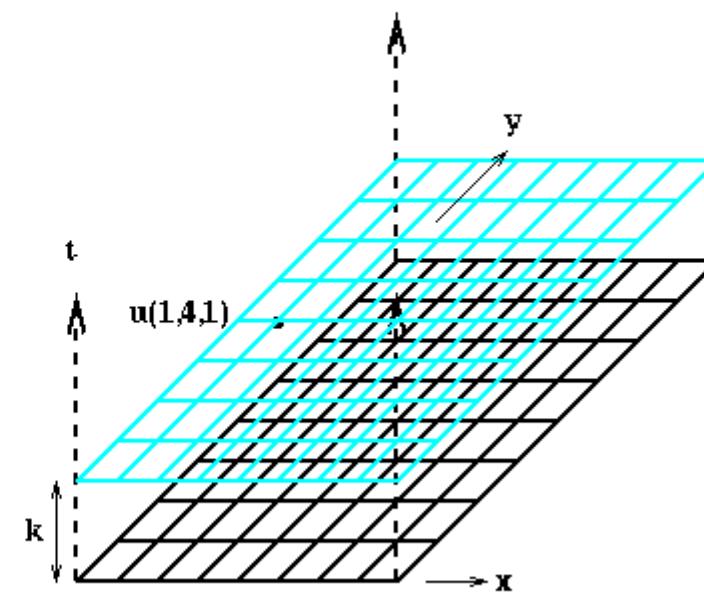
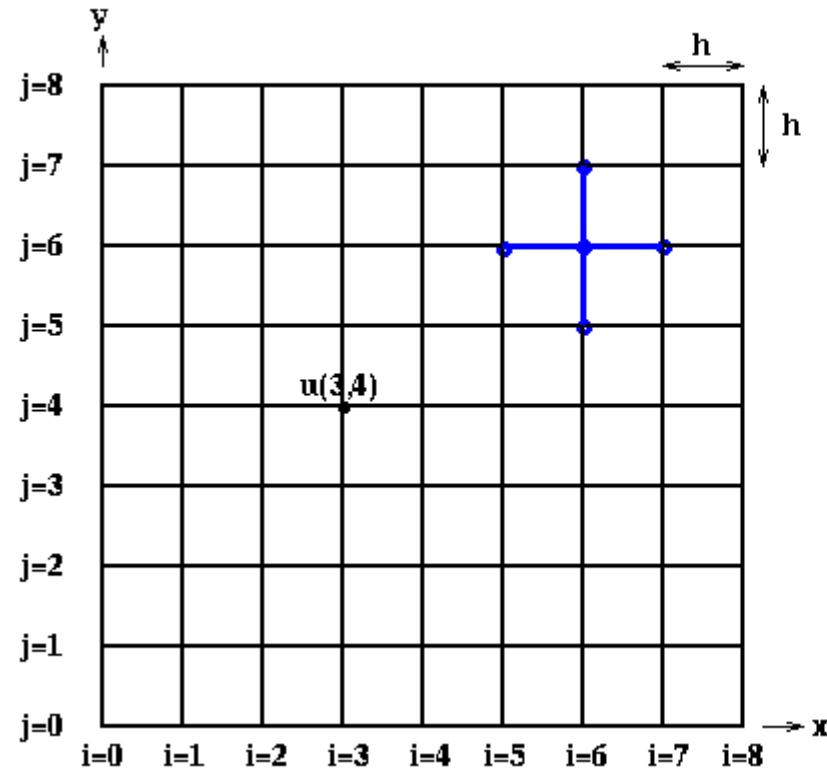


$$T_{1,3} + T_{2,4} + T_{3,3} + T_{2,2} - 4T_{2,3} = 0$$

$$T_{1,3} + 100 + T_{3,3} + T_{2,2} - 4T_{2,3} = 0$$

# Another Mesh

Discretization of the 2D Heat Equation



# Solution

## The Rest of the Equations

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0$$

$$\left( \begin{array}{cccccc} 4 & -1 & 0 & -1 & & \\ -1 & 4 & -1 & 0 & -1 & \\ 0 & -1 & 4 & 0 & 0 & -1 \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 \\ -1 & 0 & -1 & 4 & -1 & 0 & -1 \\ -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ -1 & 0 & 0 & 4 & -1 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & -1 & 0 \end{array} \right) \begin{pmatrix} T_{1,1} \\ T_{2,1} \\ T_{3,1} \\ T_{1,2} \\ T_{2,2} \\ T_{3,2} \\ T_{1,3} \\ T_{2,3} \\ T_{3,3} \end{pmatrix} = \begin{pmatrix} 75 \\ 0 \\ 50 \\ 75 \\ 0 \\ 50 \\ 175 \\ 100 \\ 150 \end{pmatrix}$$

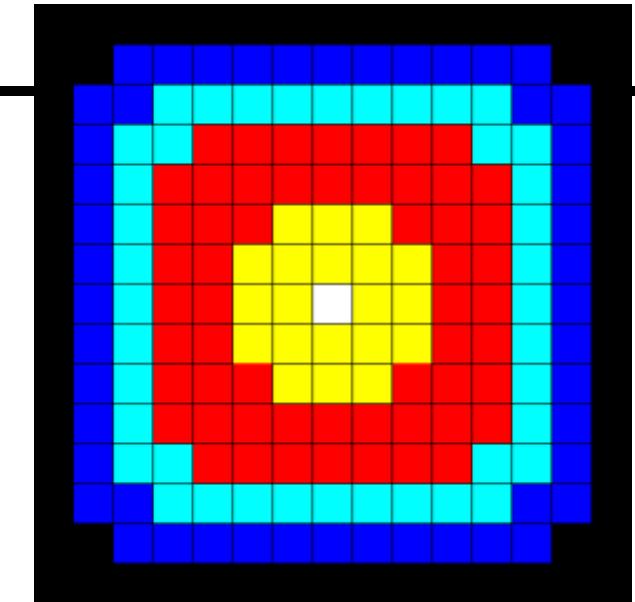
## Another evolution equation

- The calculation of an element is dependent upon neighbour element values.

$$U_{x,y} = U_{x,y} + C_x * (U_{x+1,y} + U_{x-1,y} - 2 * U_{x,y}) + C_y * (U_{x,y+1} + U_{x,y-1} - 2 * U_{x,y})$$

- A serial program like:

```
do iy = 2, ny - 1
do ix = 2, nx - 1
  u2(ix, iy) =
    u1(ix, iy) +
    cx * (u1(ix+1, iy) + u1(ix-1, iy) - 2.*u1(ix, iy)) +
    cy * (u1(ix, iy+1) + u1(ix, iy-1) - 2.*u1(ix, iy))
end do
end do
```



## Iterative Methods for Solving $\mathbf{Ax} = \mathbf{b}$

Ex:

$$(1) \quad 6x_1 - 2x_2 + x_3 = 11$$

$$(2) \quad -2x_1 + 7x_2 + 2x_3 = 5$$

$$(3) \quad x_1 + 2x_2 - 5x_3 = -1$$

$\Rightarrow$

$$x_1 = \frac{11}{6} - \frac{1}{6}(-2x_2 + x_3)$$

$$x_2 = \frac{5}{7} - \frac{1}{7}(-2x_1 + 2x_3)$$

$$x_3 = \frac{1}{5} - \frac{1}{-5}(x_1 + 2x_2)$$

$\Rightarrow$

$$x_1^{(k+1)} = \frac{1}{6}(11 - (-2x_2^{(k)} + x_3^{(k)}))$$

$$x_2^{(k+1)} = \frac{1}{7}(5 - (-2x_1^{(k)} + 2x_3^{(k)}))$$

$$x_3^{(k+1)} = \frac{1}{-5}(-1 - (x_1^{(k)} + 2x_2^{(k)}))$$

**Initial Approximation:**  $x_1 = 0, x_2 = 0, x_3 = 0$

Iter	0	1	2	3	4	$\dots$	8
$x_1$	0	1.833	2.038	2.085	2.004	$\dots$	2.000
$x_2$	0	0.714	1.181	1.053	1.001	$\dots$	1.000
$x_3$	0	0.2	0.852	1.080	1.038	$\dots$	1.000

Stop when  $\| \vec{x}^{(k+1)} - \vec{x}^{(k)} \| < 10^{-4}$

Need to define **norm**  $\| \vec{x}^{(k+1)} - \vec{x}^{(k)} \|$ .

## Jacobi Method for $\mathbf{Ax} = \mathbf{b}$

$$x_i^{k+1} = \frac{1}{a_{ii}}(b_i - \sum_{j \neq i} a_{ij}x_j^k) \quad i = 1, \dots, r$$

Parallel

### Jacobi method in a matrix-vector form

$$x^{k+1} =$$

Example:

or in general

$$(1) \quad 6x_1 - 2x_2 + x_3 = 11$$

$$(2) \quad -2x_1 + 7x_2 + 2x_3 = 5$$

$$(3) \quad x_1 + 2x_2 - 5x_3 = -1$$

$\Rightarrow$

$$x_1 = \frac{11}{6} - \frac{1}{6}(-2x_2 + x_3)$$

$$x_2 = \frac{5}{7} - \frac{1}{7}(-2x_1 + 2x_3)$$

$$x_3 = \frac{1}{5} - \frac{1}{5}(x_1 + 2x_2)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^{k+1} = \begin{bmatrix} 0 & \frac{2}{6} & -\frac{1}{6} \\ \frac{2}{7} & 0 & -\frac{2}{7} \\ \frac{1}{5} & \frac{2}{5} & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^k + \begin{pmatrix} \frac{11}{6} \\ \frac{5}{7} \\ \frac{1}{5} \end{pmatrix}$$

Parallel solution:

- Distribute rows
- Perform computation on owner-computes
- Perform all-gather after each iteration

## SPMD Code for $y = H * x + d$

```
me=mynode();
for i = 1 to n do
    if proc_map(i) == me, then do  $S_i$ :
         $S_i$  :     $y[i] = 0;$ 
         $i1 = Local(i)$ 
        for j = 1 to n do
             $y[i1] = y[i1] + a[i1][j] * x[j] + d[i1]$ 
        endfor
    endfor
endfor
```

**Optimized solution:** write the Jacobi method as:

```
Repeat
    For  $i = 1$  to  $n$ 
         $y_i^{new} = 0.5(y_{i-1}^{old} + y_{i+1}^{old} - h^2)$ 
    Endfor
Until  $\| \vec{y}^{new} - \vec{y}^{old} \| < \varepsilon$ 
```

## The SOR method

SOR (Successive Over Relaxation).

The rate of convergence can be improved (accelerated) by the SOR method:

**Step 1:** Use the Gauss-Seidel Method.

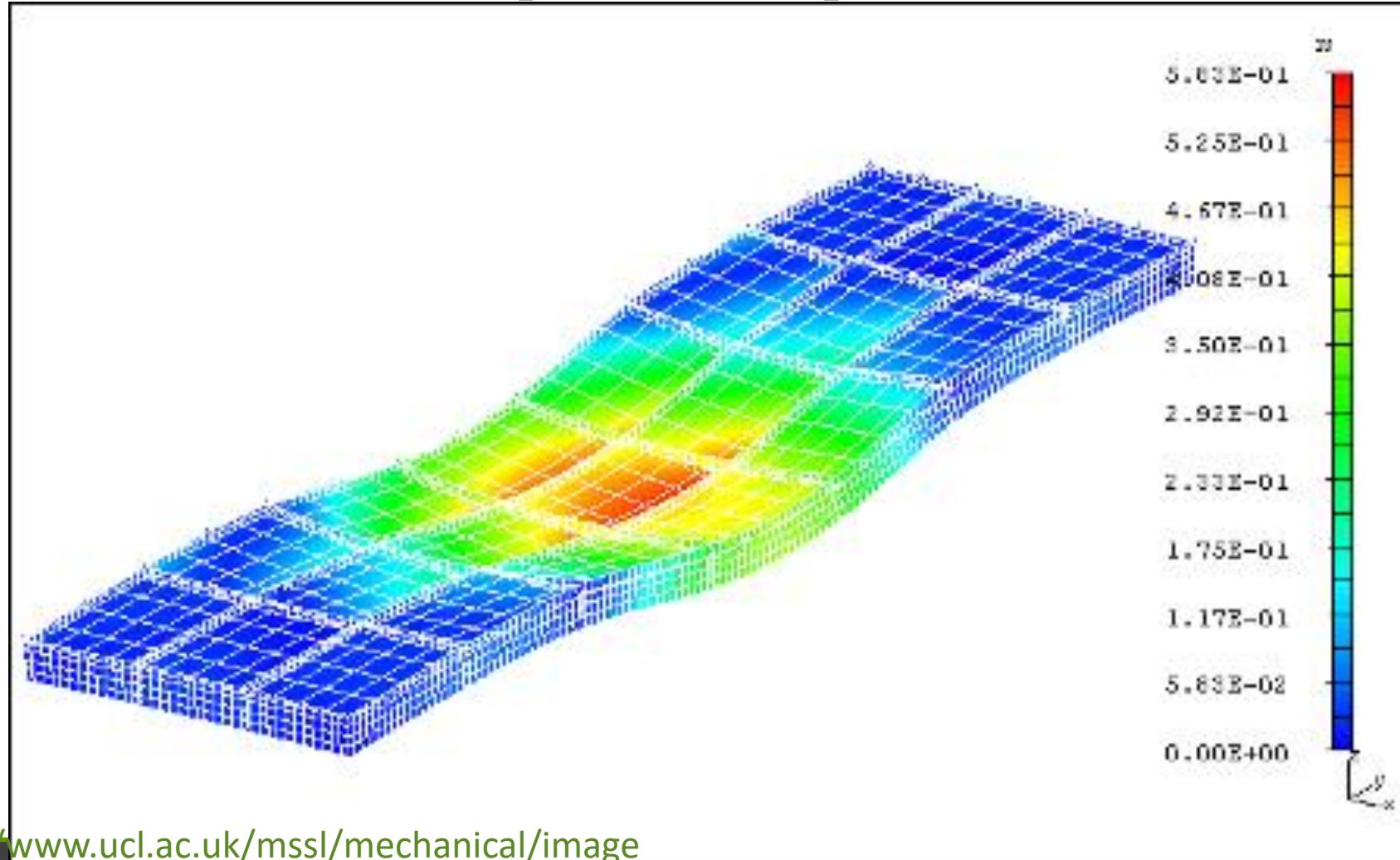
$$x^{k+1} = Hx^k + d$$

**Step 2:**

$$x^{k+1} = x^k + w(x^{k+1} - x^k)$$

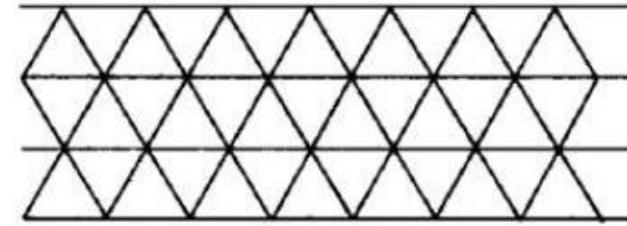
# Weather forecasting? – FEM with similar idea

## □ FEM: Finite Element Model [有限元分析]

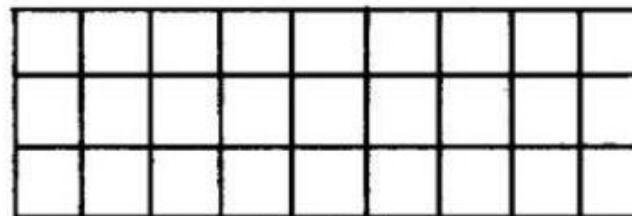


<https://www.ucl.ac.uk/mssl/mechanical/images/analysis/Mode02A.png>

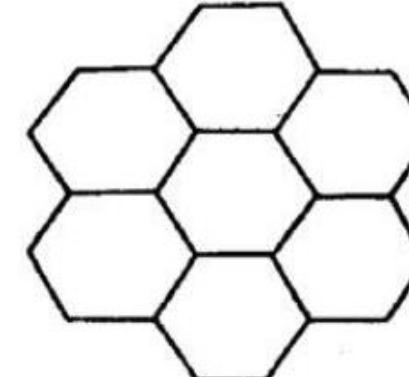
# We need Super Computer/HPC



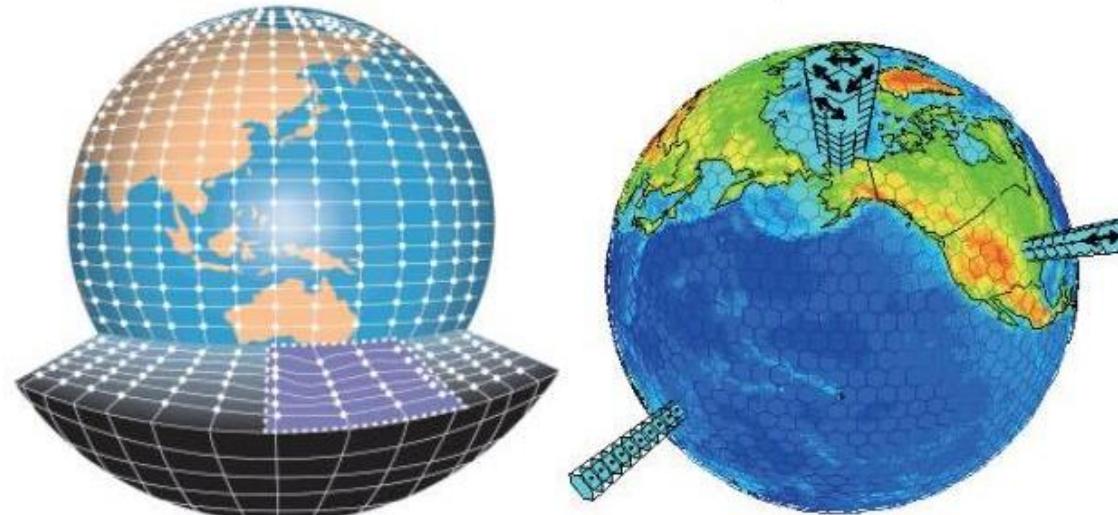
(a) 正三角形网络



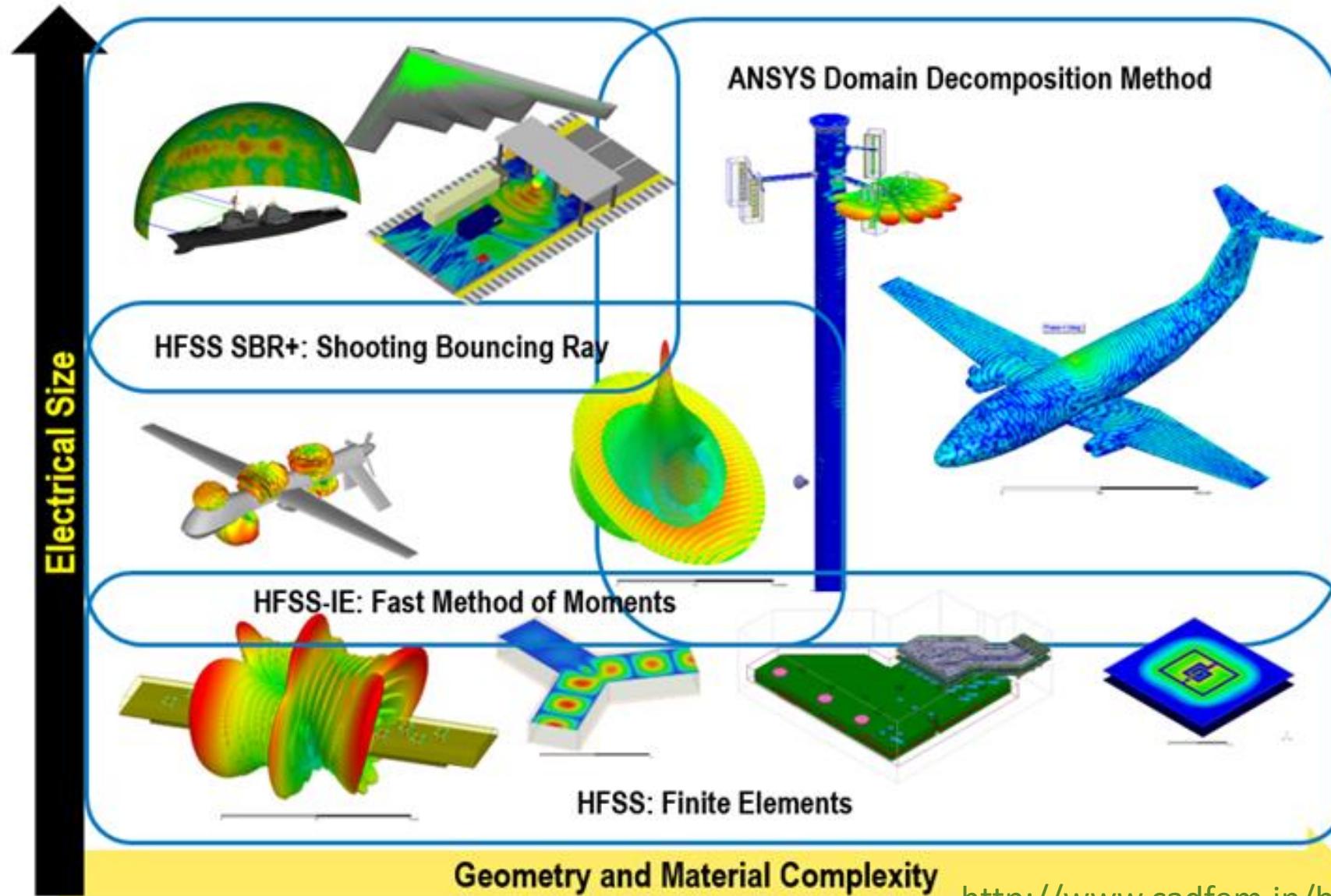
(b) 正方形网络



(c) 正六角形网络

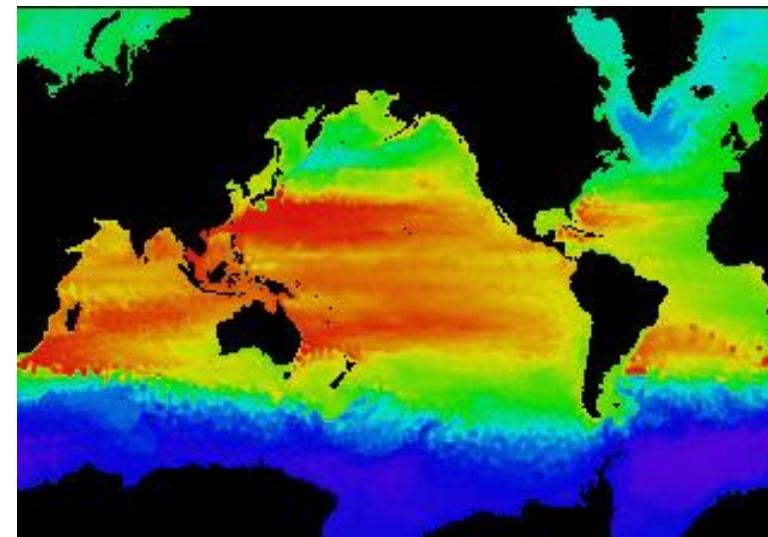


# How to model is the KEY



# Global Climate Modeling Problem

- Problem is to compute:  
 $f(\text{latitude, longitude, elevation, time}) \rightarrow \text{“weather”} = (\text{temperature, pressure, humidity, wind velocity})$
- Approach:
  - *Discretize* the domain, e.g., a measurement point every 10 km
  - Devise an algorithm to predict weather at time  $t + \delta t$  given  $t$
- Uses:
  - Predict major events, e.g., El Nino
  - Use in setting air emissions standards
  - Evaluate global warming scenarios



Source: <http://www.epm.ornl.gov/chammp/chammp.html>

# Global Grid (0.25\*0.25 → 1440X720 dots = 62208000)

---

- If we take resolution as 0.25\*0.25, and 60 vertical layers, the matrix size will be **1440\*720\*60 = 6.22\*10<sup>7</sup>**.
- If 6 variables, the number is **6\*6.22\*10<sup>7</sup>**.
- If we want to predict the weather in 24 hours, and 5 minutes as span, we should compute values of
  - **288\*6\*6.22\*10<sup>7</sup>. ≈ 1.08\*10<sup>11</sup>**.
  - If you want to predict future 3 days, 5 days, 10 days?
- So **HUGE!** We need more powerful computers!
  - Super Computers, HPC etc.

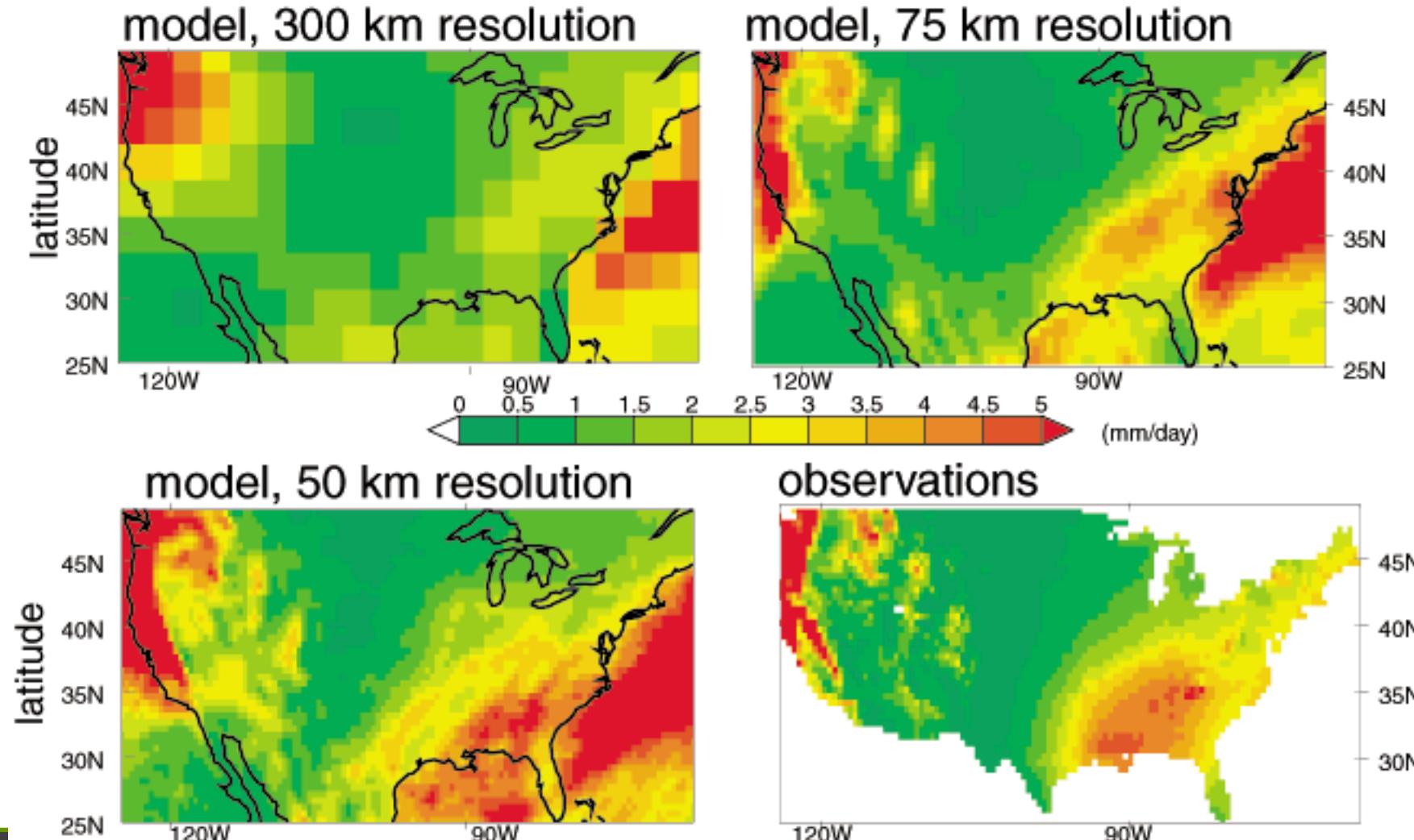
# Global Climate Modeling Computation

---

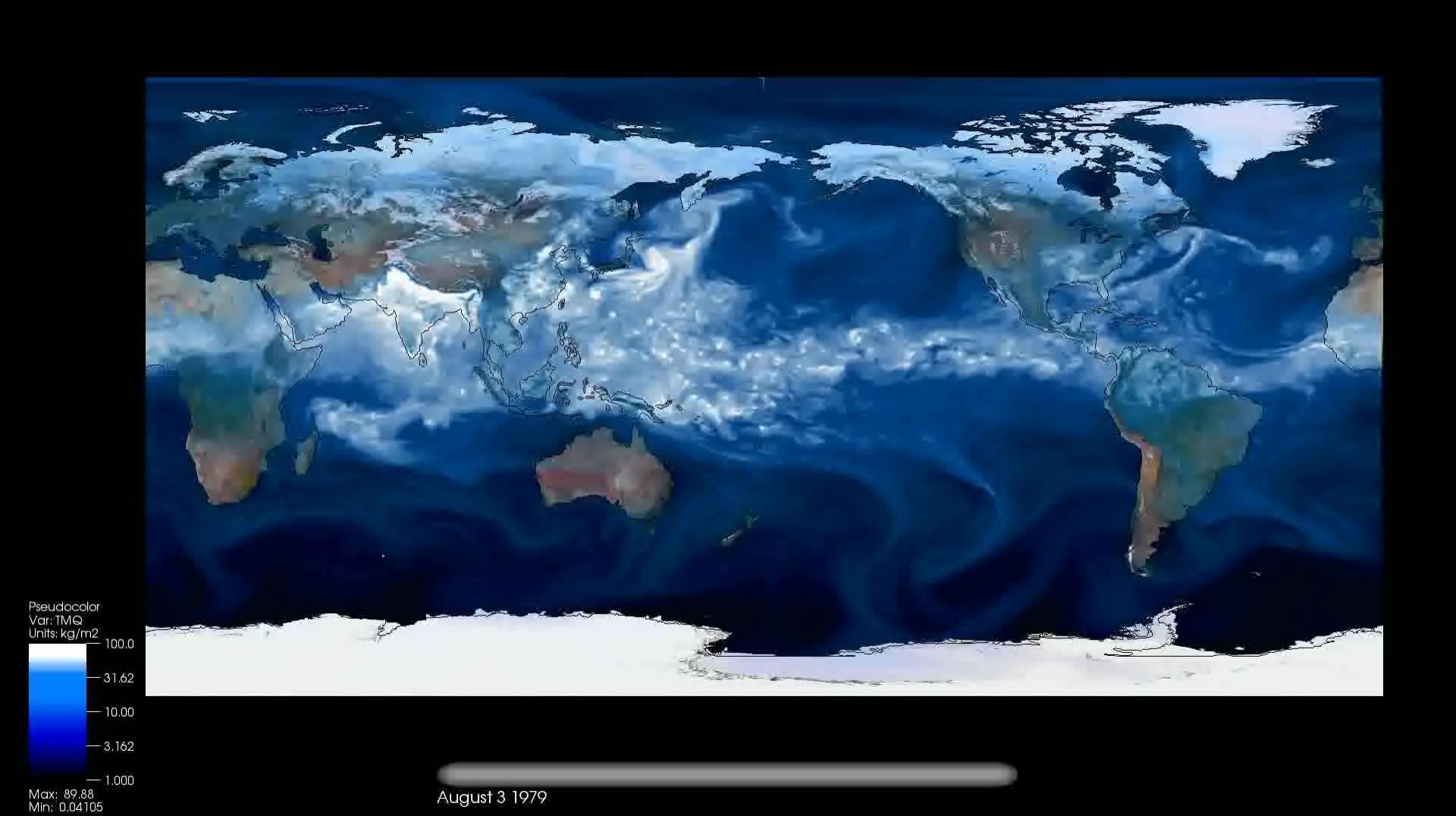
- One piece is modeling the fluid flow in the atmosphere
  - Solve Navier-Stokes equations
    - Roughly 100 Flops per grid point with 1 minute timestep
- Computational requirements:
  - To match real-time, need  $5 \times 10^{11}$  flops in 60 seconds = 8 Gflop/s
  - Weather prediction (7 days in 24 hours)  $\rightarrow$  56 Gflop/s
  - Climate prediction (50 years in 30 days)  $\rightarrow$  4.8 Tflop/s
  - To use in policy negotiations (50 years in 12 hours)  $\rightarrow$  288 Tflop/s
- To double the grid resolution, computation is 8x to 16x
- State of the art models require integration of atmosphere, clouds, ocean, sea-ice, land models, plus possibly carbon cycle, geochemistry and more
- Current models are coarser than this

## Wintertime Precipitation

As model resolution becomes finer, results converge towards observations

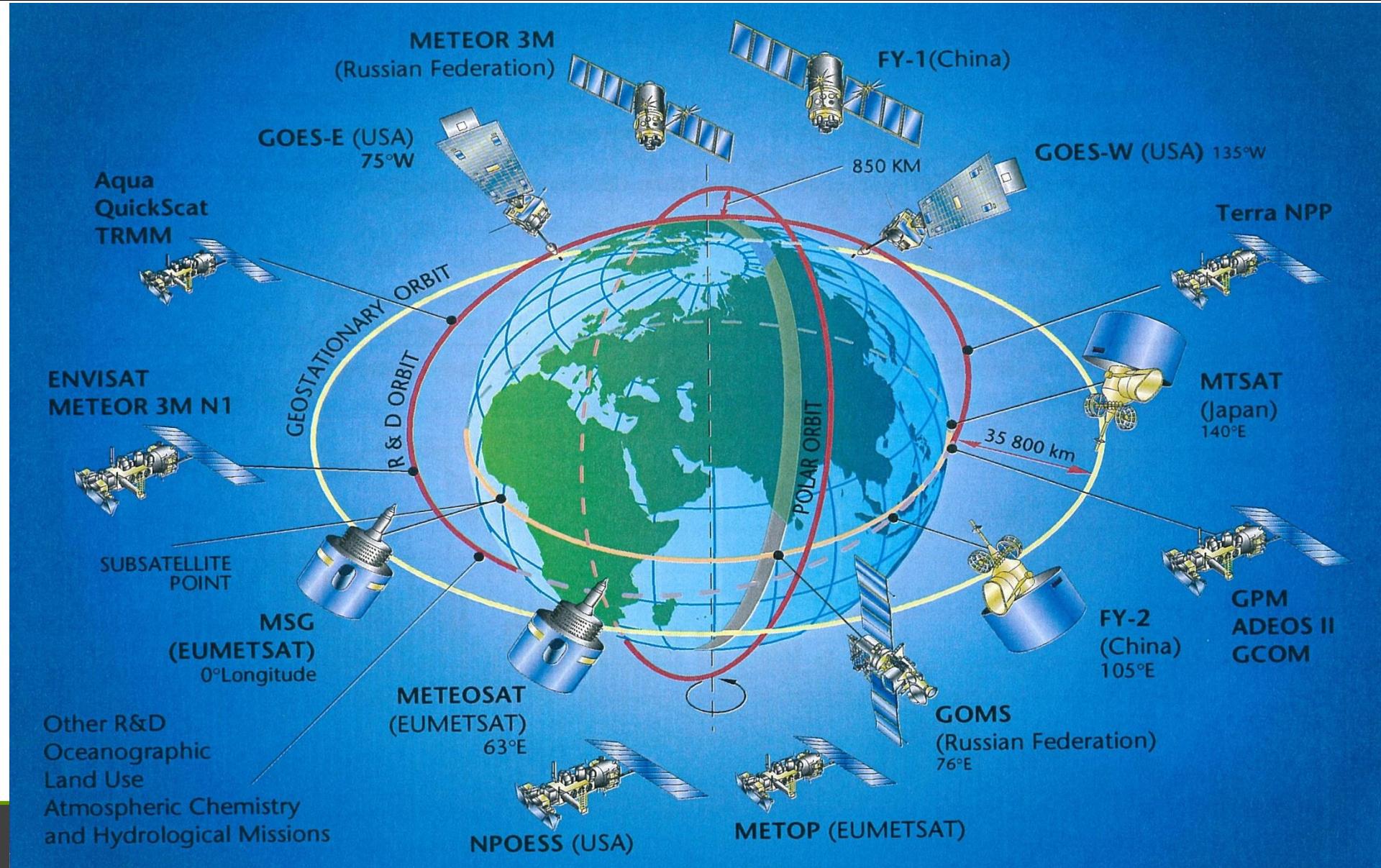


# U.S.A. Hurricane

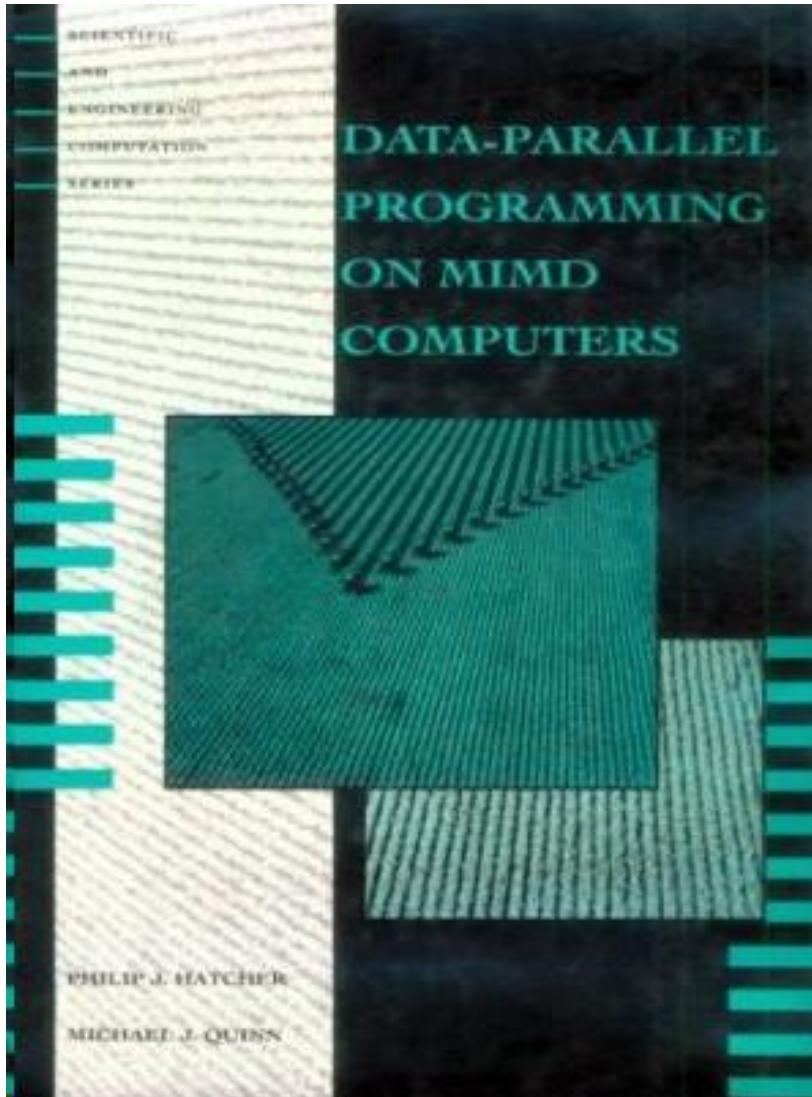


Source: Data from M.Wehner, visualization by Prabhat, LBNL

# Meteorological Satellites



# Michael J. Quinn



- Data-Parallel Programming on MIMD Computers
- Philip J. Hatcher, Michael J. Quinn
- 1991

The results we report in this book are largely due to the efforts of many graduate students at the University of New Hampshire and Oregon State University. Charles A. Grasso implemented the first generic host (*ghost*) program for the nCUBE™ 3200. Jeffrey E. F. Friedl modified the University of Virginia's Very Portable C Compiler to generate code for the nCUBE node processors, developed the valuable UNIX™-to-nCUBE and nCUBE-to-UNIX binary file conversion programs, and implemented the second *ghost* program for the nCUBE. Karen C. Jourdenais designed and implemented our first Dataparallel C compiler for the nCUBE. Lutz H. Hamel built the second-generation Dataparallel C compiler for the nCUBE and ported the GNU C compiler to generate code for the nCUBE node processors. Robert R. Jones built the front end

那时候的学生呀！现在，大数据开源，我们也应该引入高校，让学生改着玩！

Michael J. Quinn

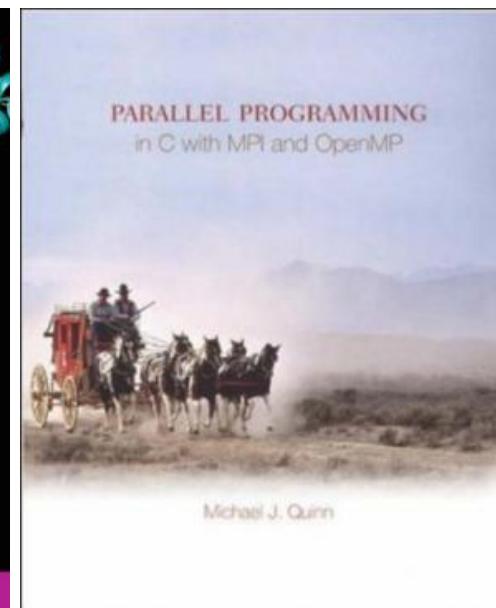
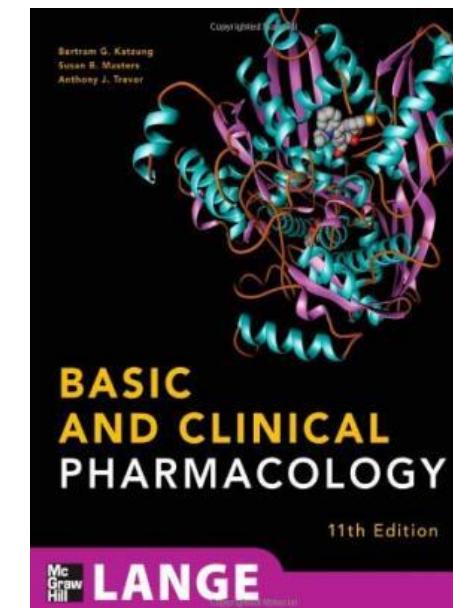


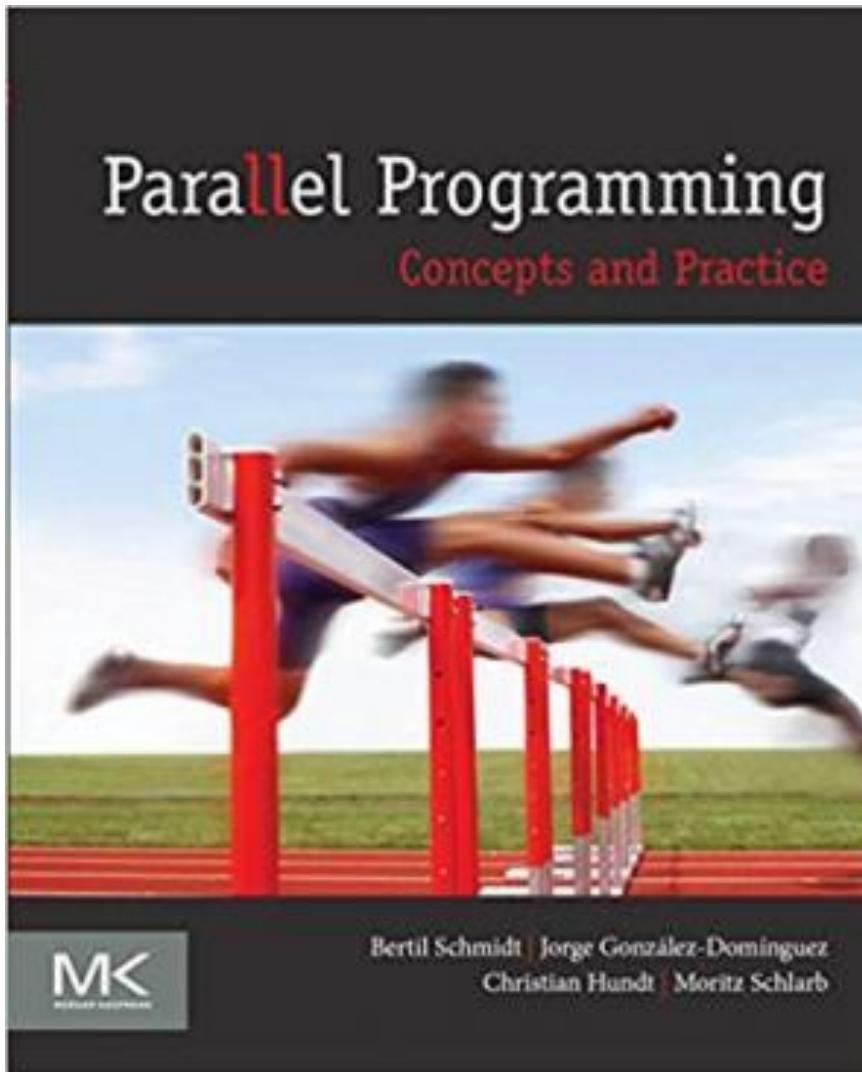
- > 第1章动机和历史
- > 第2章并行体系结构
- > 第3章并行算法设计
- > 第4章消息传递编程
- > 第5章Eratosthenes筛法
- > 第6章Floyd算法
- > 第7章性能分析
- > 第8章矩阵向量乘法
- > 第9章文档分类
- > 第10章蒙特卡洛法
- > 第11章矩阵乘法
- > 第12章线性方程组求解
- > 第13章有限差分方法
- > 第14章排序
- > 第15章快速傅立叶变换
- > 第16章组合搜索
- > 第17章共享存储编程
- > 第18章融合OpenMP和MPI
- > 附录A MPI函数

□ Parallel programming in C with MPI and Open MP

□ Michael J Quinn

□ 2003



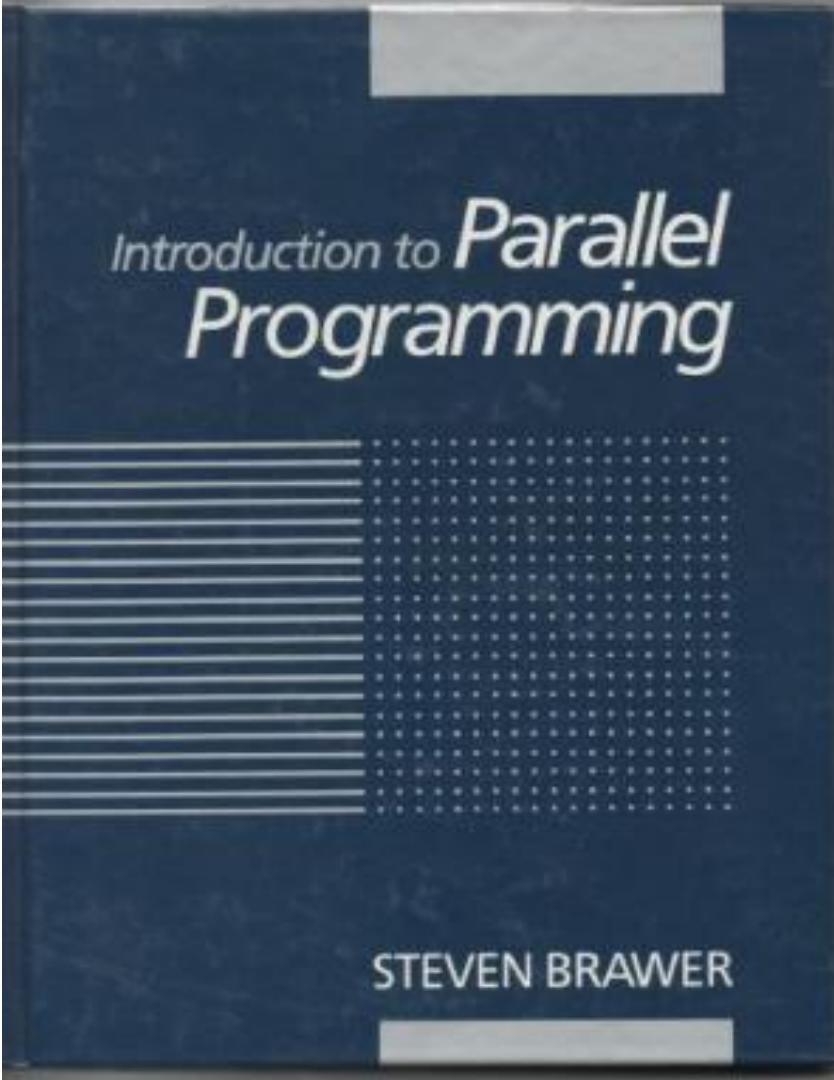


- Parallel Programming: Concepts and Practice
- Bertil Schmidt, Jorge González-Domínguez, Christian Hundt, Moritz Schlarb

## □ 2017 CONSIDERATIONS WHEN DESIGNING PARALLEL PROGRAMS

Assume you are given a problem or a sequential code that needs to be parallelized. Independent of the particular architecture or programming language that you may use, there are a few typical considerations that need to be taken into account when designing a parallel solution:

- **Partitioning:** The given problem needs to be decomposed into pieces. There are different ways how to do this. Important examples of partitioning schemes are data parallelism, task parallelism, and model parallelism.
- **Communication:** The chosen partitioning scheme determines the amount and types of required communication between processes or threads.
- **Synchronization:** In order to cooperate in an appropriate way, threads or processes may need to be synchronized.
- **Load balancing:** The amount of work needs to be equally divided among threads or processes in order to balance the load and minimize idle times.

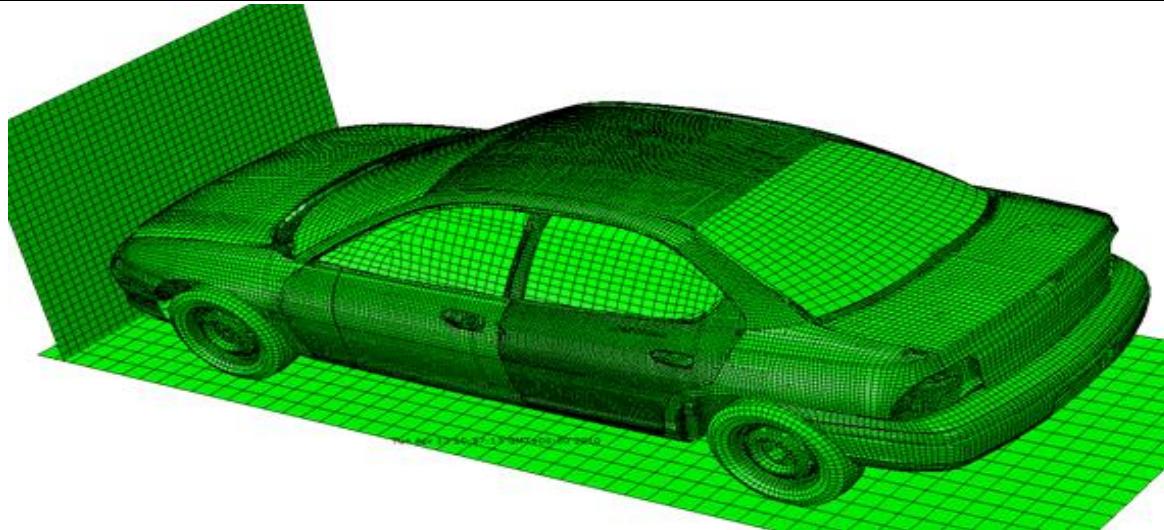


Introduction to **Parallel**  
**Programming**

STEVEN BRAWER

- Introduction to Parallel Programming
- Steven Brawer (Auth.)
- 1989

# 案例1：某汽车设计研究院整车碰撞分析



正面碰撞仿真模型

- 128核大规模并行、网格规模（千万级网格）；
- **Ls-dyna**、Abaqus；
- 仿真应用：汽车碰撞仿真，内容包括正碰、侧碰、偏置碰、多车追尾碰撞等；
- 将原本在PC机上需要运行2-3天的单个计算模型的计算时间成功的缩短到了几个小时；

# LS-DYNA

---

- LS-DYNA is a general-purpose finite element program capable of simulating complex real world problems. It is used by the automobile, aerospace, construction, military, manufacturing, and bioengineering industries. LS-DYNA is optimized for shared and distributed memory Unix, Linux, and Windows based, platforms, and it is fully QA'd by LSTC. The code's origins lie in highly nonlinear, transient dynamic finite element analysis using explicit time integration.

<https://www.lstc.com/products/ls-dyna>

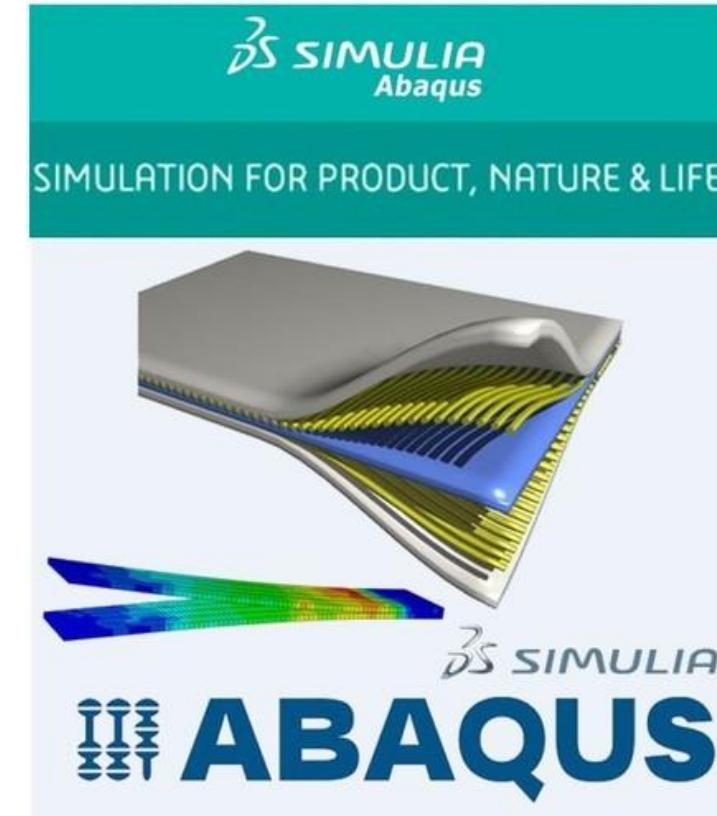


# Abaqus

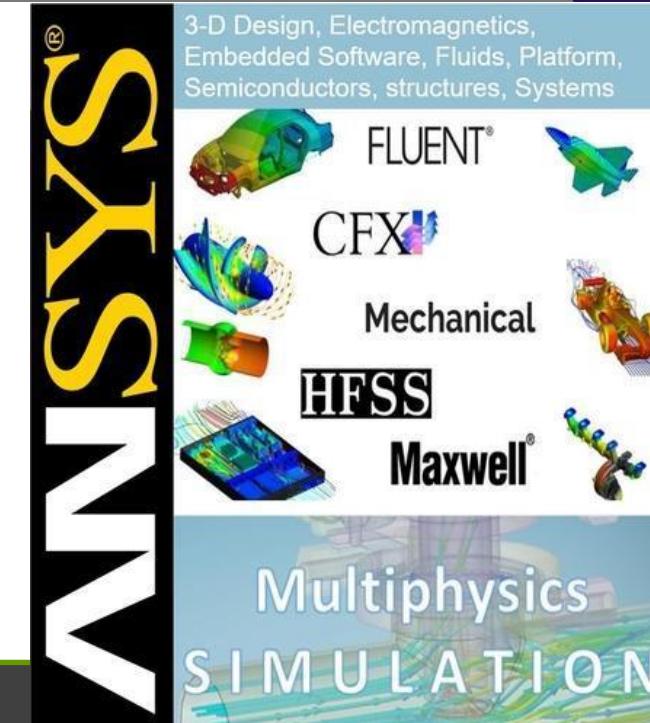
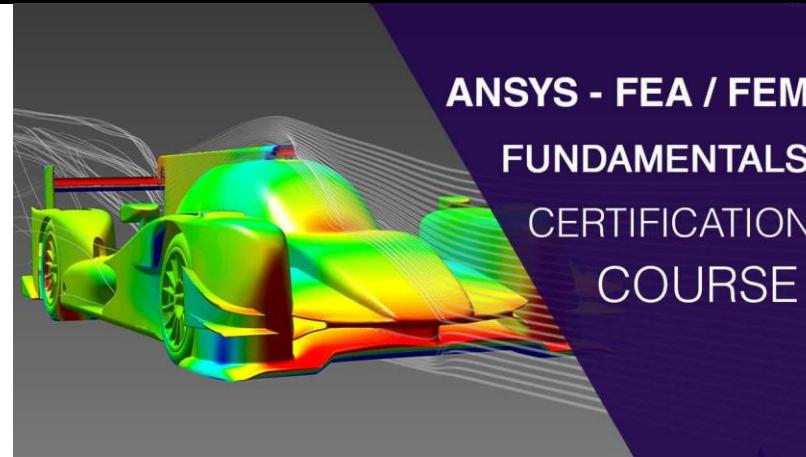
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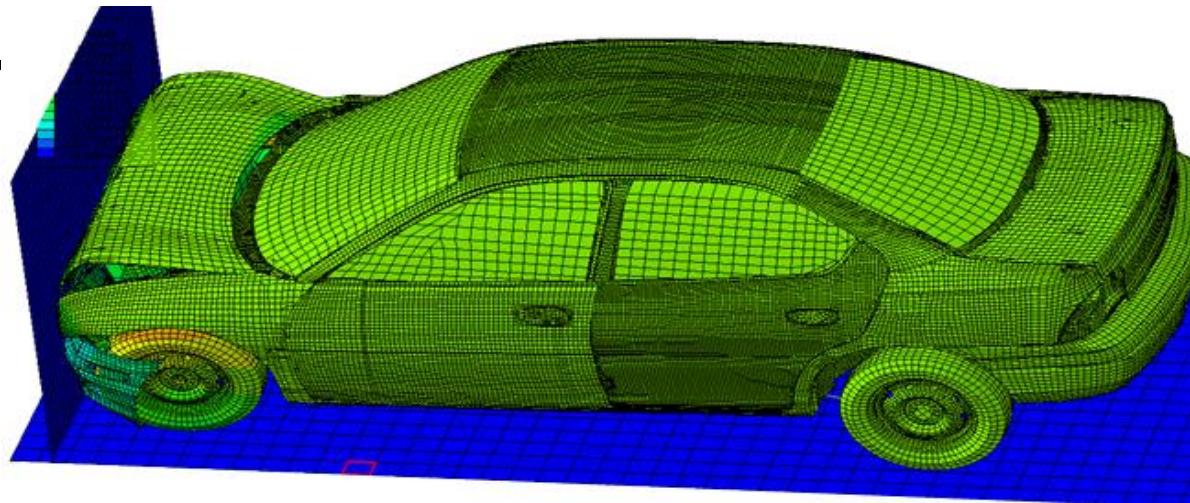
- The Abaqus Unified FEA product suite offers powerful and complete solutions for both routine and sophisticated engineering problems covering a vast spectrum of industrial applications. In the automotive industry engineering work groups are able to consider full vehicle loads, dynamic vibration, multibody systems, impact/crash, nonlinear static, thermal coupling, and acoustic-structural coupling using a common model data structure and integrated solver technology.

[https://www.3ds.com/products-  
services/simulia/products/abaqus/](https://www.3ds.com/products-services/simulia/products/abaqus/)

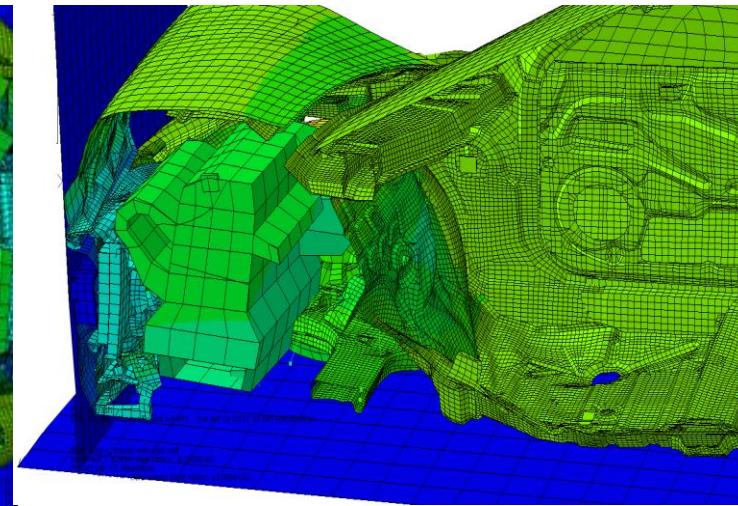
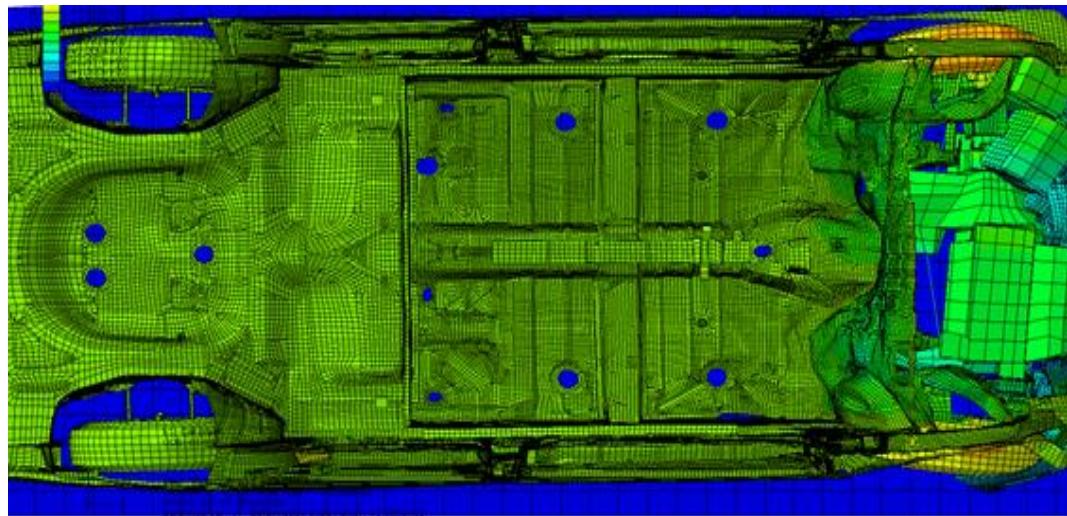


- If you've ever seen a rocket launch, flown on an airplane, driven a car, used a computer, touched a mobile device, crossed a bridge, or put on wearable technology, chances are you've used a product where ANSYS software played a critical role in its creation. ANSYS is the global leader in engineering simulation. We help the world's most innovative companies deliver radically better products to their customers



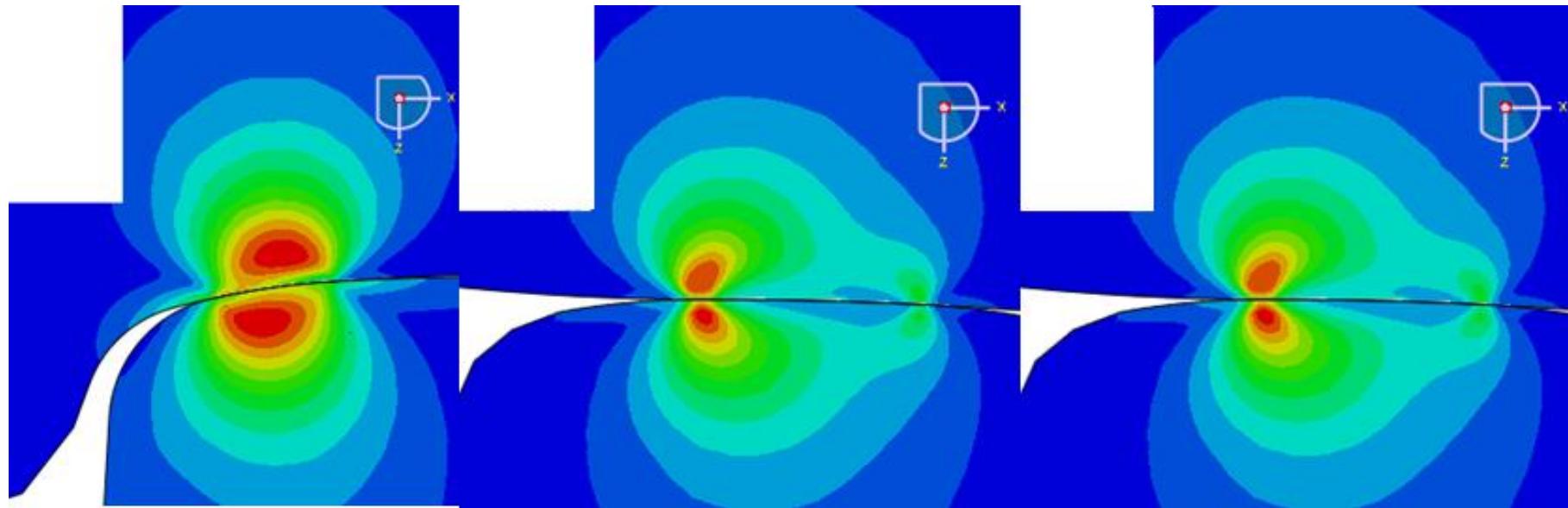


仿真结果：整车碰撞



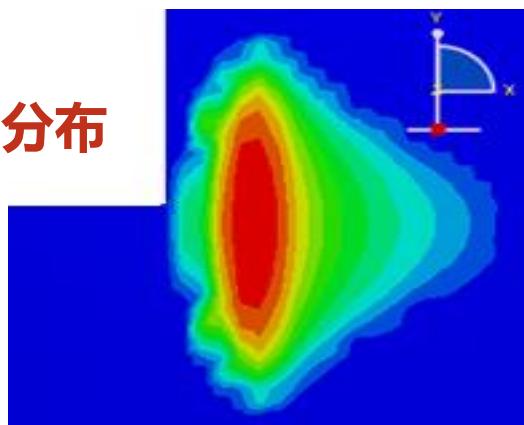
仿真结果：车体内部零件损伤变形

# 案例2：铁科院轮轨接触优化设计



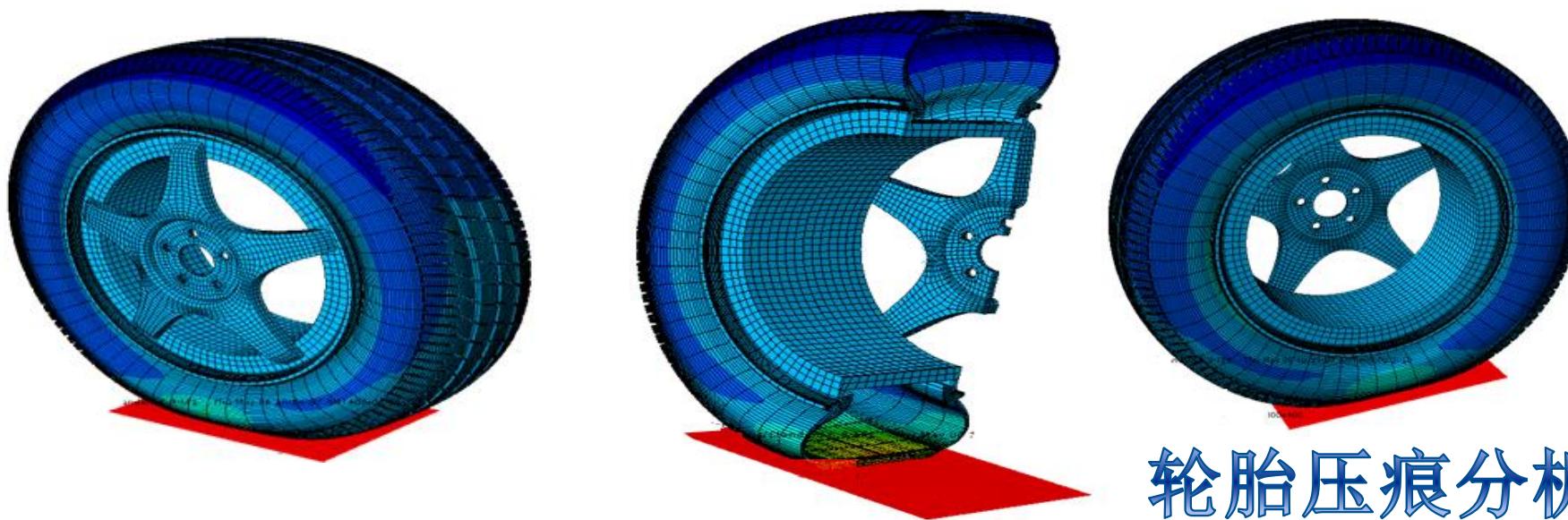
接触应力云图

接触压力分布



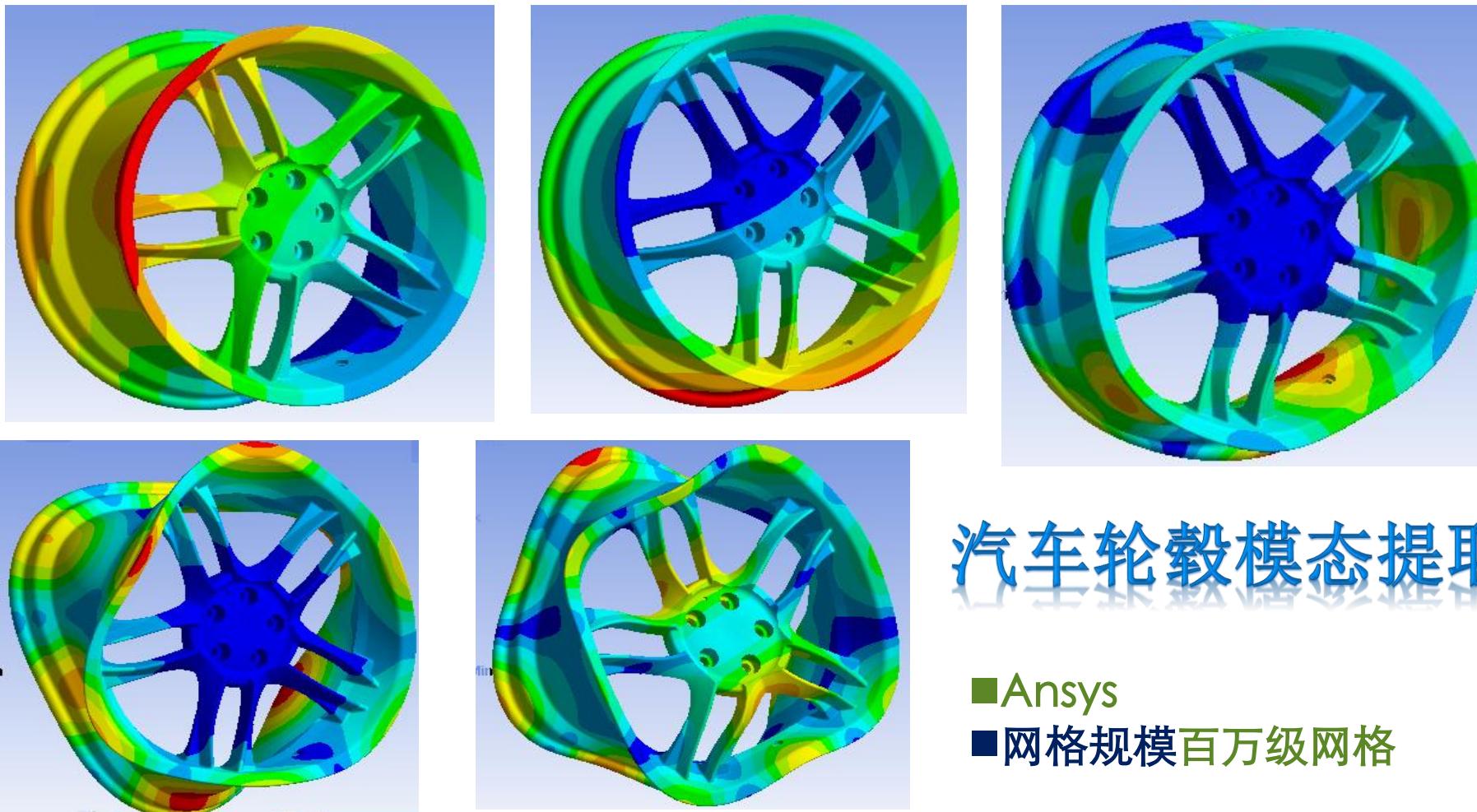
- 128核大规模并行、网格规模（千万级网格）；
- Abaqus；
- 完成数十个仿真模型计算，作业规模从百万网格到上亿网格不等；

# 案例3：某汽车设计研究院汽车轮胎性能优化设计



轮胎压痕分析

- 在工业云平台上完成汽车轮毂模态提取、汽车轮毂强度校核、轮胎压痕分析
- 128核大规模并行；
- Abaqus、Ansys；
- 整个仿真项目历时数月，先后提交23个计算模型，完成数十个仿真模型计算，作业规模从百万网格到千万级网格不等；



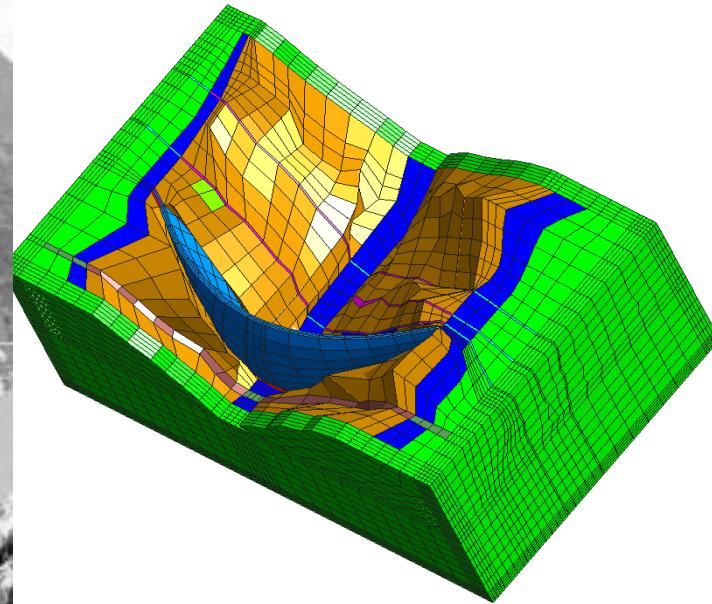
## 汽车轮毂模态提取

- Ansys
- 网格规模百万级网格

## 案例4：小湾拱坝地震响应分析



小湾拱坝实景图片

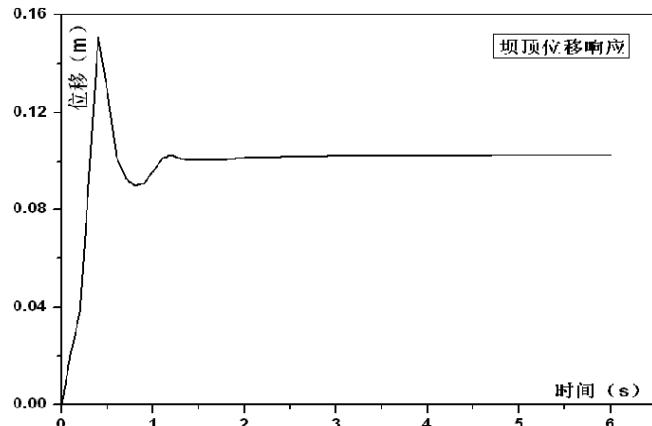


有限元造型

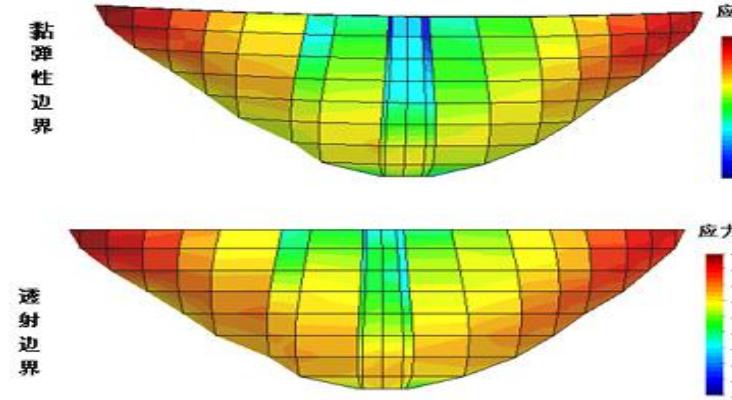


小湾拱坝实景图片

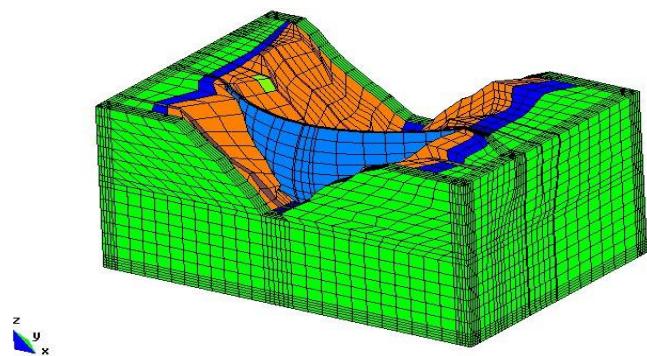
■共有28285个节点，1075个接触点对，总共23862个单元（其中23022个8节点六面体单元，736个六节点五面体棱柱单元，90个四边形边界单元，14个三角形边界单元），25种材料。



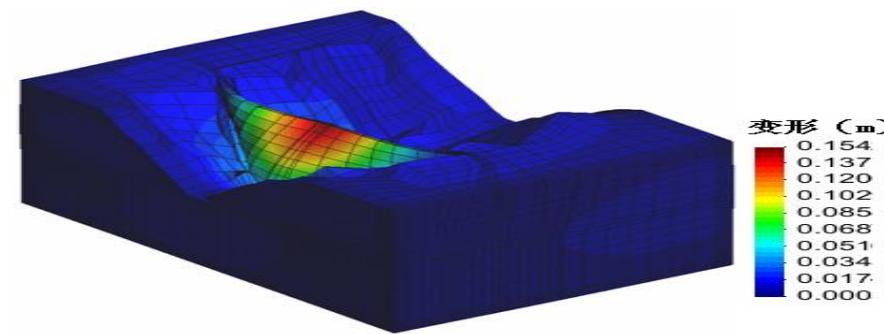
坝顶节点在坝体重力作用下的稳定过程



小湾坝体地震荷载作用下上游面压应力分布

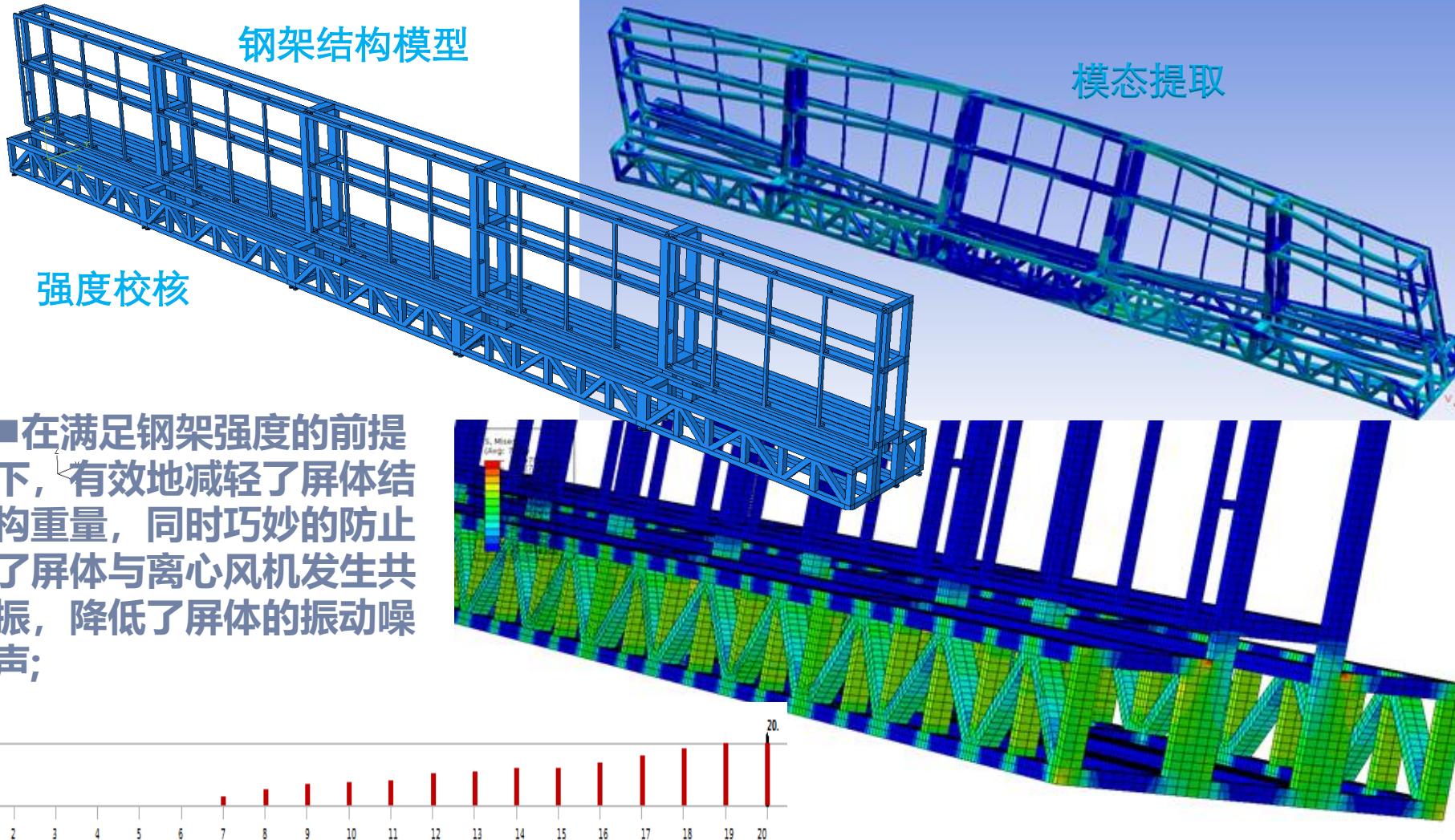


小湾拱坝计算模型



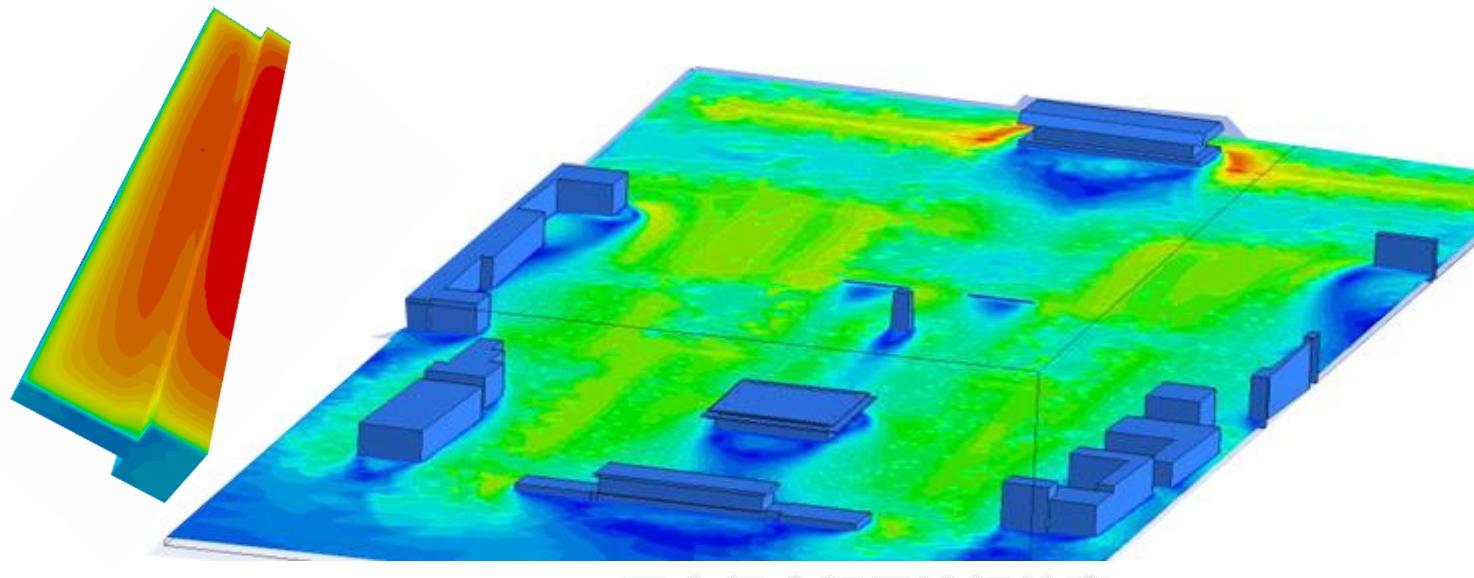
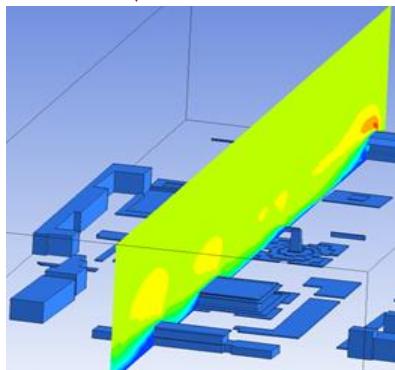
小湾拱坝造型整体静位移

# 案例5：天安门广场LED屏钢架结构优化设计



# 案例6：天安门广场LED屏抗风载性能设计

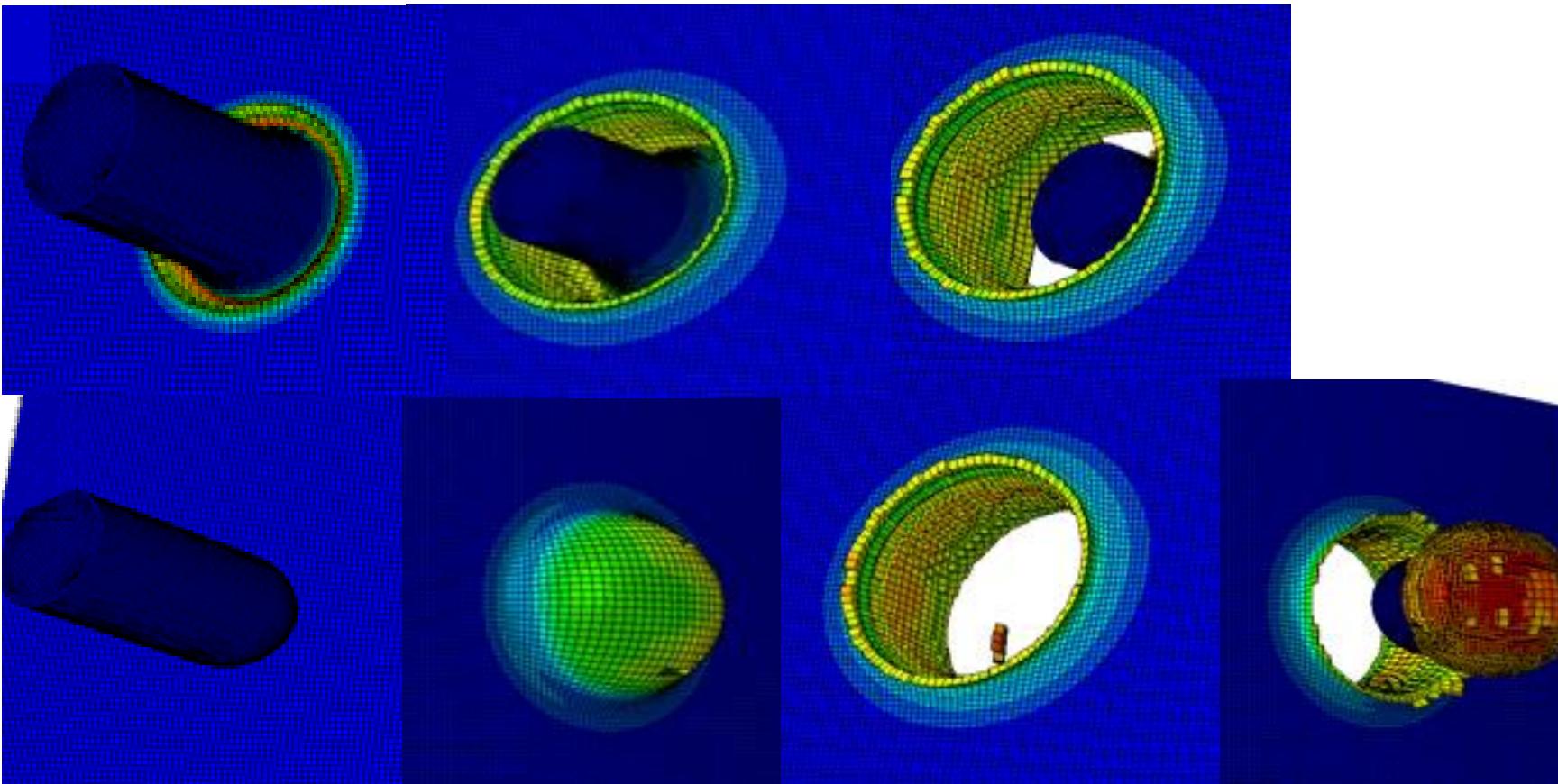
LED屏表面风压分布



■采用Fluent软件模拟天安门广场风速场分布，计算出LED屏表面风压分布情况。将得到的风压结果作为边界条件附加到Abaqus计算模型上，进一步评测风载对屏体钢架结构的影响，确定出LED屏的最大抗风载能力；

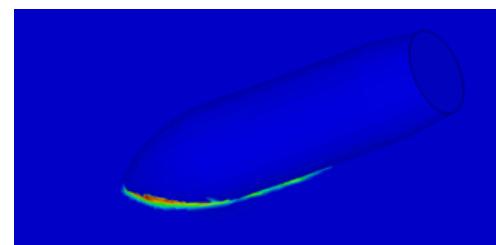
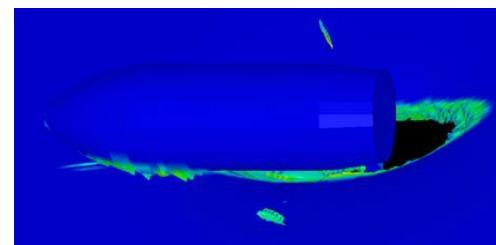
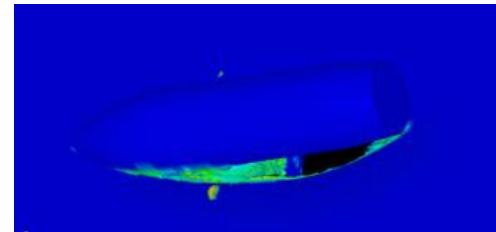
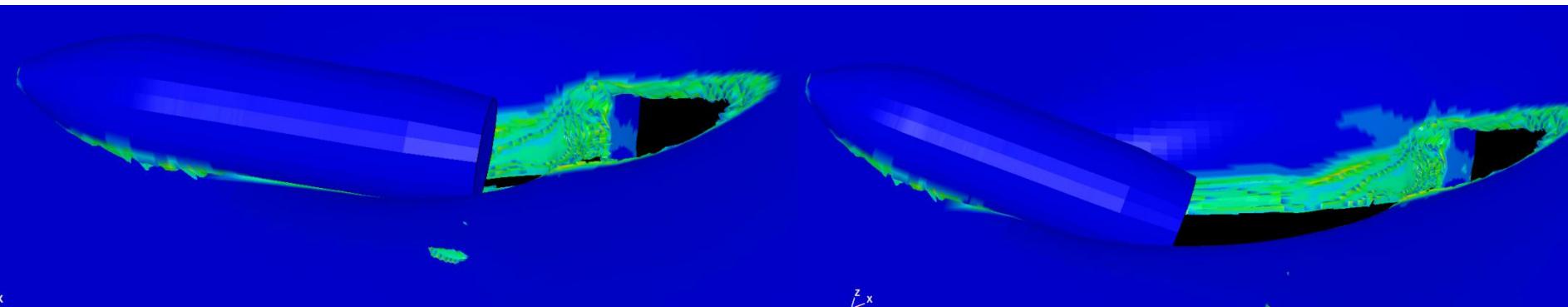
# 案例7:某兵工厂弹体侵彻分析

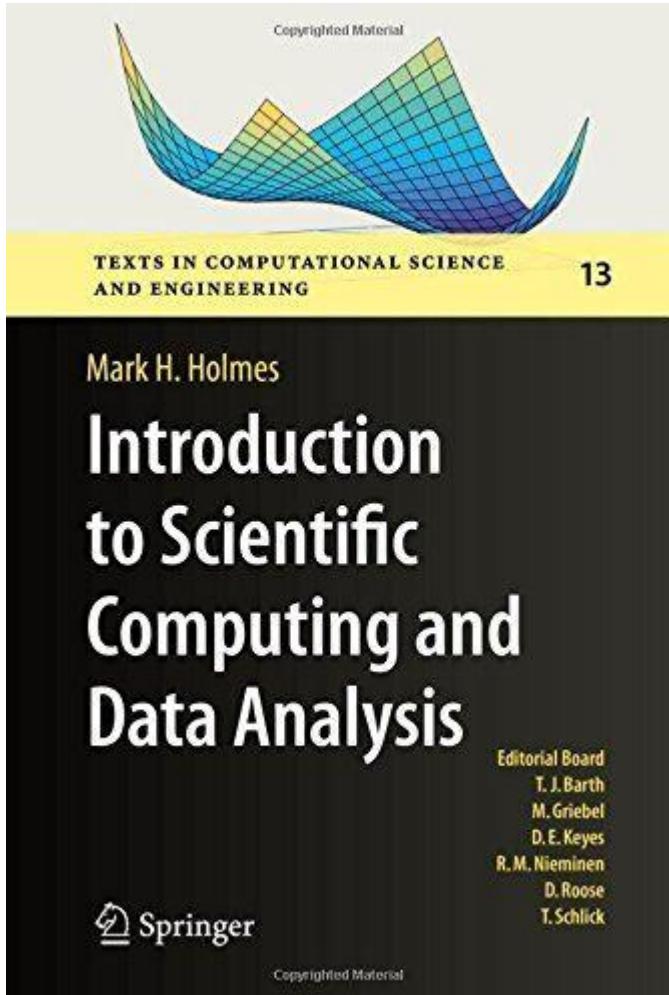
- 在模拟弹体侵彻芳纶、钢板、陶瓷等材料的侵彻过程,有效减少打靶实验次数;
- 采用软件Abaqus\ls-dyna;
- 网格规模(百万网格);



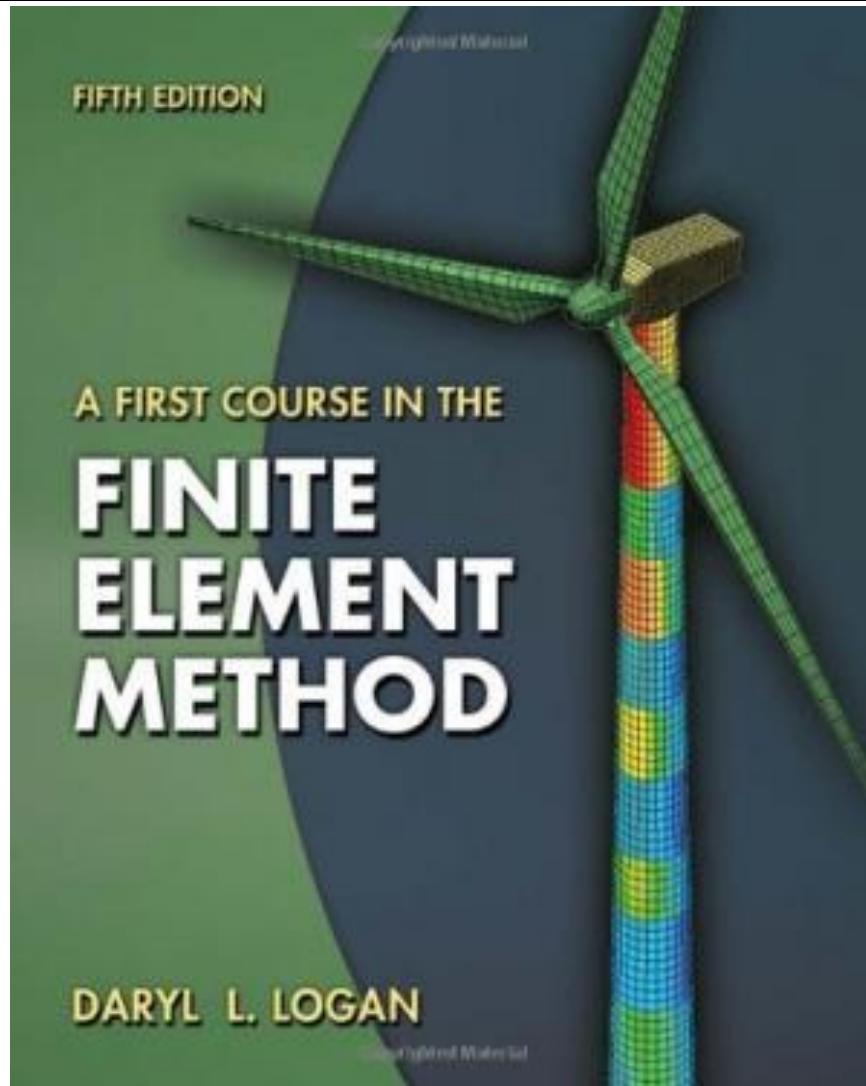
# 案例8:某兵工厂弹体破甲性能分析

- 在主要用于某型坦克前甲板抗弹性能改良;
- 采用软件Ls-dyna;
- 网格规模(百万网格);
- 通过破甲性能分析,得出甲板加强肋的最佳设计方案,有效提高了某型坦克前甲板的抗弹性能;

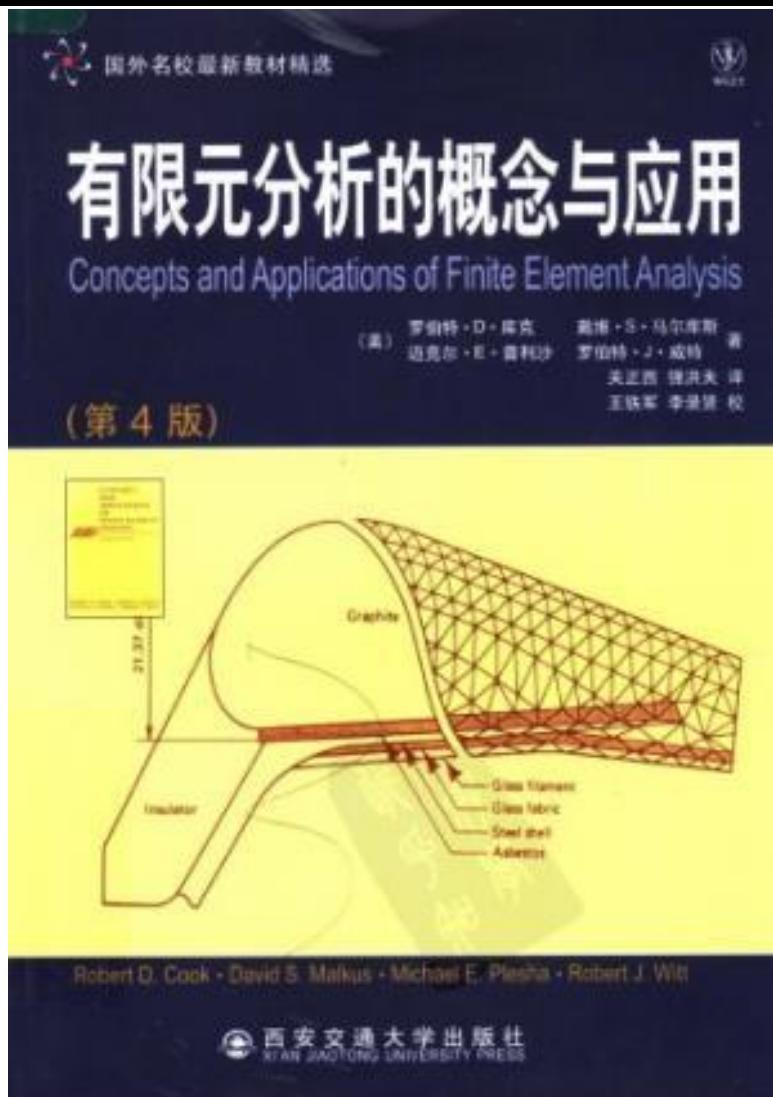




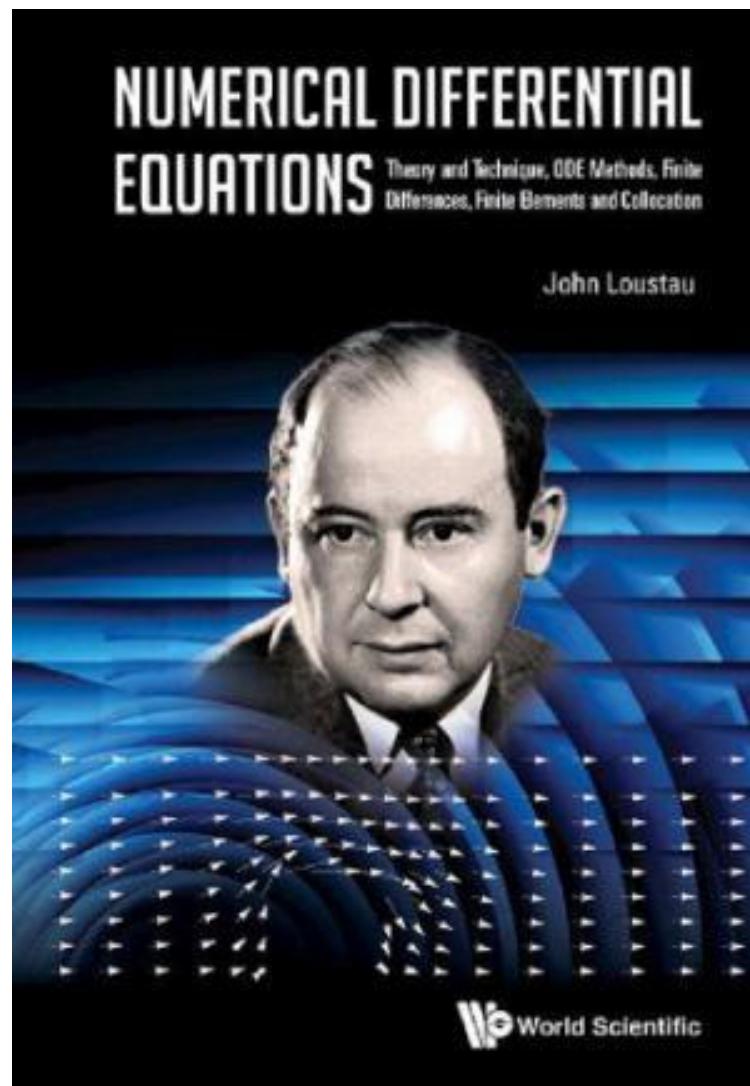
- **Introduction to Scientific Computing and Data Analysis**
- **Authors** **Mark H. Holmes (auth.)**
- **Year** **2016**
- **Pages** **505**
- **Publisher** **Springer International Publishing**
- **Language** **en**
- **ISBN** **9783319302546**



- A First Course in the Finite Element Method
- Daryl L. Logan
  - A FIRST COURSE IN THE FINITE ELEMENT METHOD provides a simple, basic approach to the course material that can be understood by both undergraduate and graduate students without the usual prerequisites (i.e. structural analysis). The book is written primarily as a basic learning tool for the undergraduate student in civil and mechanical engineering whose main interest is in stress analysis and heat transfer. The text is geared toward those who want to apply the finite element method as a tool to solve practical physical problems.



- 有限元分析的概念与应用-第四版
- Robert D. Cook, David S. Malkus, Michael E. Plesha, Robert J. Witt
- 《有限元分析的概念与应用(第4版)》适合机械、力学、土木、动力、材料、水利和航空航天等专业高年级本科生和研究生作为有限元课程的教材及教学参考书，对相关专业的工程技术人员和科研工作者也有很好的参考价值。基础理论和工程应用并重的特色鲜明，是一本有限元课程的优秀教材。



- Numerical Differential Equations: Theory and Technique, ODE Methods, Finite Differences, Finite Elements and Collocation
- John Loustau
  - This text presents numerical differential equations to graduate (doctoral) students. It includes the three standard approaches to numerical PDE, FDM, FEM and CM, and the two most common time stepping techniques, FDM and Runge-Kutta. We present both the numerical technique and the supporting theory.

The applied techniques include those that arise in the present literature. The supporting mathematical theory includes the general convergence theory. This material should be readily accessible to

OXFORD APPLIED MATHEMATICS  
AND COMPUTING SCIENCE SERIES

**Numerical Solution of  
Partial Differential  
Equations: Finite  
Difference Methods**

G.D. SMITH

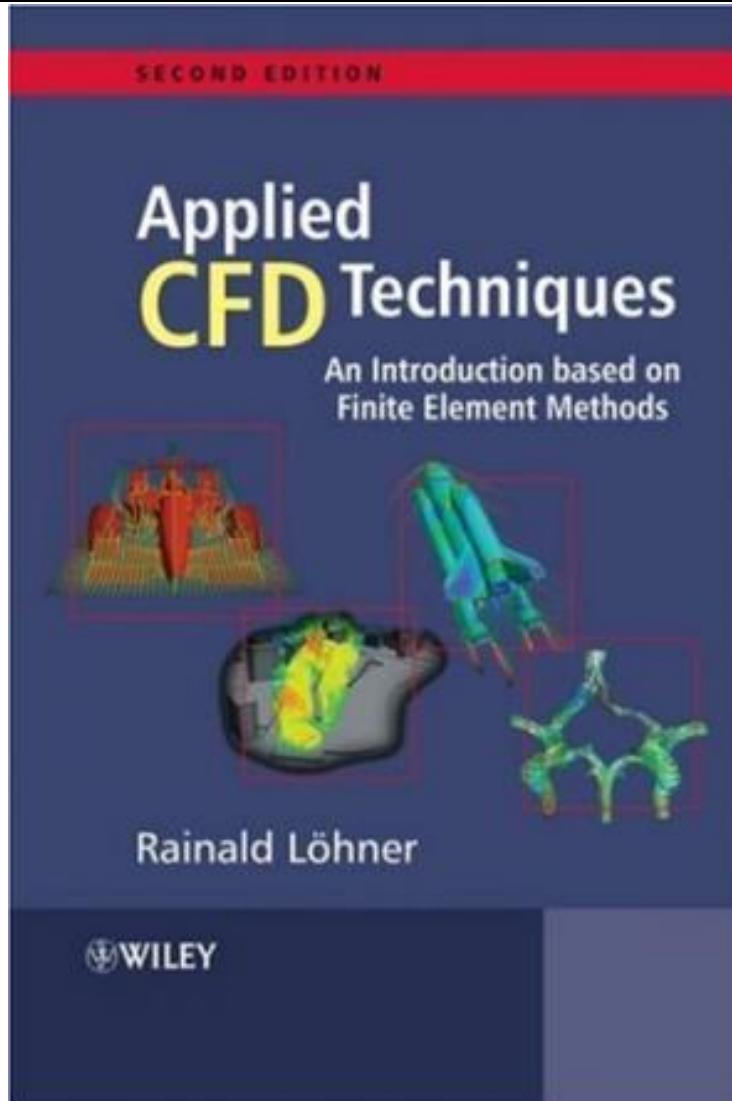
Third Edition



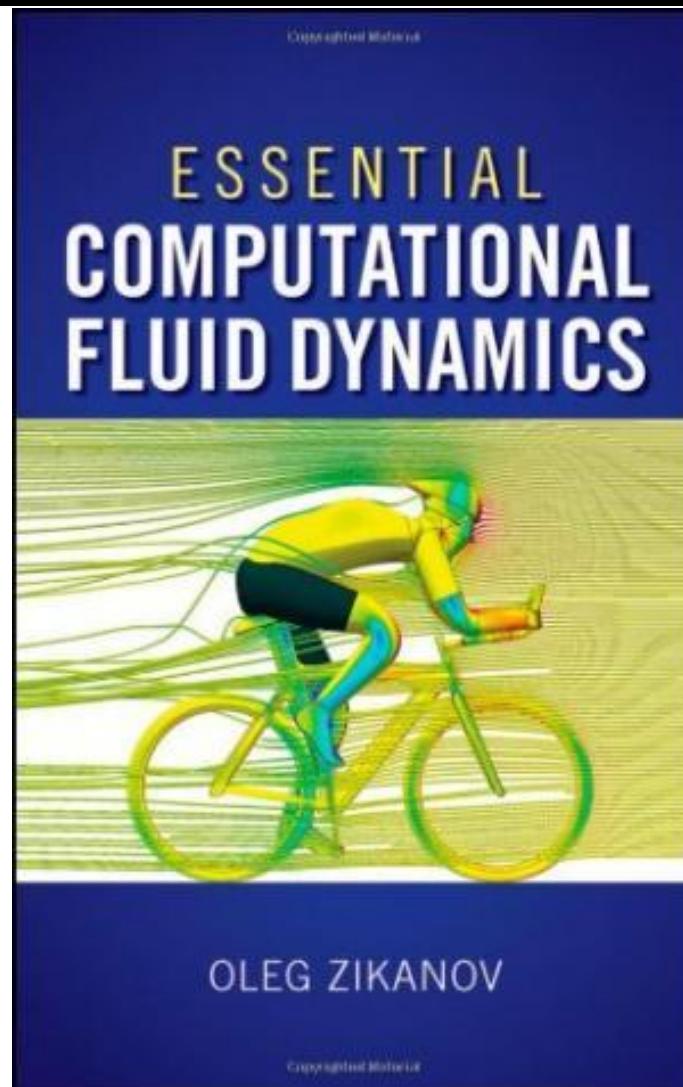
□ **Numerical Solution of Partial Differential Equations:  
Finite Difference Methods**

□ **G. D. Smith**

■ Substantially revised, this authoritative study covers the standard finite difference methods of parabolic, hyperbolic, and elliptic equations, and includes the concomitant theoretical work on consistency, stability, and convergence. The new edition includes revised and greatly expanded sections on stability based on the Lax-Richtmeyer definition, the application of Pade approximants to systems of ordinary differential equations for parabolic and hyperbolic equations, and a considerably improved presentation of iterative methods. A fast-paced introduction to numerical methods, this will be a useful volume for students of mathematics and



- Applied computational fluid dynamics techniques: an introduction based on finite element methods
- Prof Rainald Löhner
  - Computational fluid dynamics (CFD) is concerned with the efficient numerical solution of the partial differential equations that describe fluid dynamics. CFD techniques are commonly used in the many areas of engineering where fluid behavior is an important factor. Traditional fields of application include aerospace and automotive design, and more recently, bioengineering and consumer and medical electronics. With Applied Computational Fluid Dynamics Techniques, 2nd edition, Rainald Löhner introduces the reader to the techniques required to achieve efficient CFD solvers, forming a bridge between basic theoretical and algorithmic



- Essential Computational Fluid Dynamics
- Oleg Zikanov
  - This book serves as a complete and self-contained introduction to the principles of Computational Fluid Dynamic (CFD) analysis. It is deliberately short (at approximately 300 pages) and can be used as a text for the first part of the course of applied CFD followed by a software tutorial. The main objectives of this non-traditional format are: 1) To introduce and explain, using simple examples where possible, the principles and methods of CFD analysis and to demystify the 'black box' of a CFD software tool, and 2) To provide a basic understanding of how CFD problems are set and which factors affect the success and failure of the analysis. Included in the text are the mathematical and physical foundations of CFD, formulation of CFD

# Chapter 2: HPC with an example

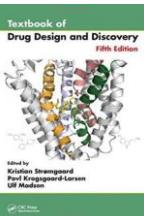
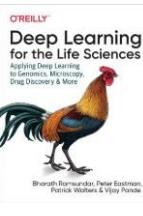
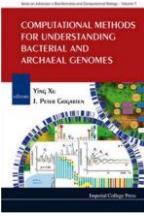
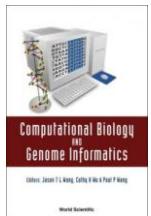
## ❑ Faster for larger data

- Many problems/applications need HPComputers
  - Weather / Climate, Cryptography, Nuclear Weapons Design, Scientific Simulation, Petroleum Exploration, Aerospace Design, Automotive Design, Pharmaceutical Design, Data Mining, Data Assimilation
- Heat dynamics as an example to understand Numeric Computing/Scientific Computing
  - Like Finite Element Analysis (有限元分析)
- Other examples

# Many applications need more computation power

## With **HUGE** data to be processed

- Scientific computing, business applications etc.



OAK RIDGE NATIONAL LABORATORY

Government-Classified Work

Government - Research

(Severe) Weather Prediction & Climate Modeling



Instituto Nacional de Meteorología

España



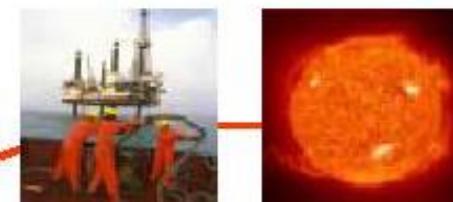
Automotive Design & Safety



Drug Discovery & Genomic Research



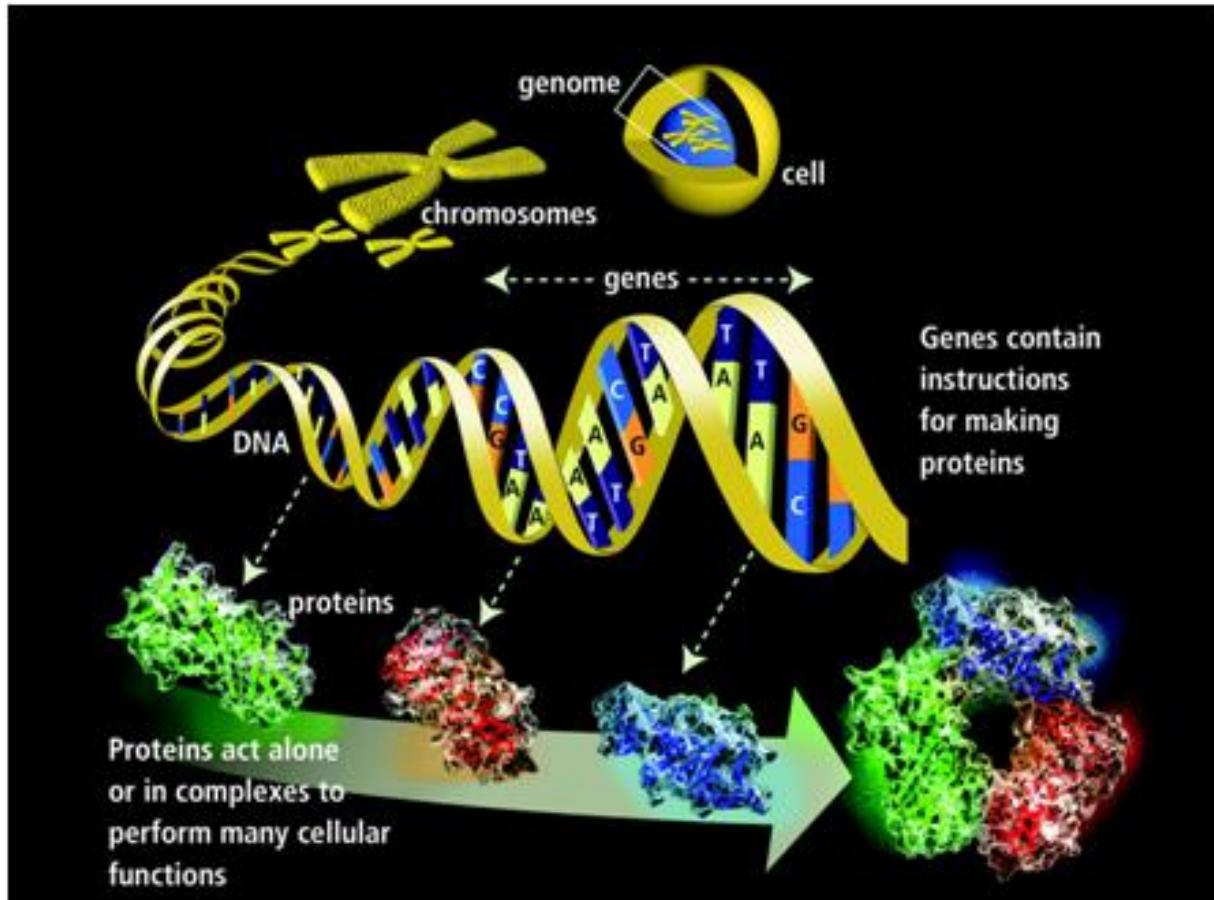
Aircraft/Spacecraft Design & Fuel-Efficiency



Oil Exploration & Energy Research

Basic Scientific Research





**DNA sequence**

↓

**Protein sequence**

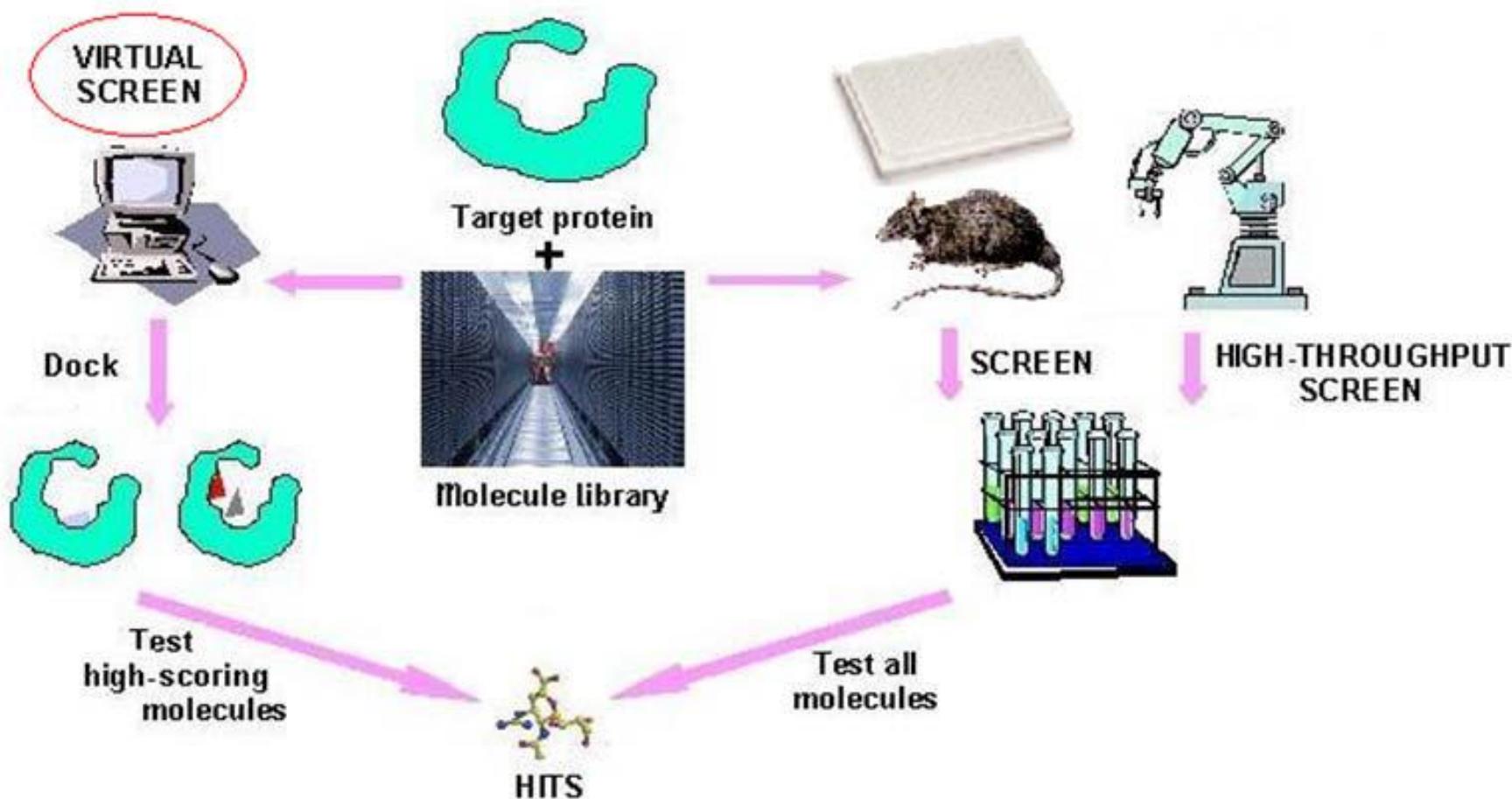
↓

**Protein structure**

↓

**Protein function**

# Virtual screening in drug discovery



# 用矩阵和连通表



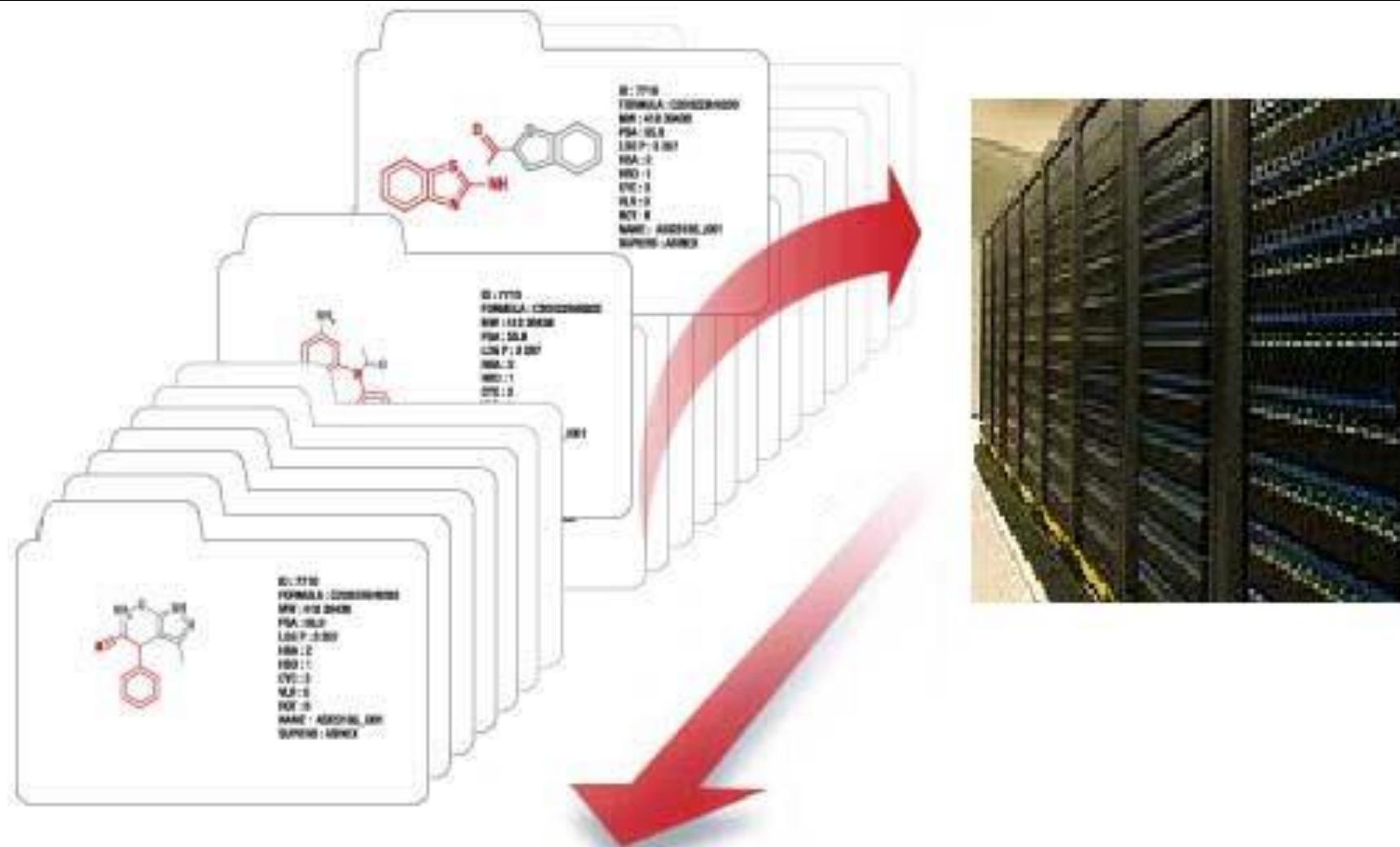
Representation		Name	
Caffeine		Common Name	
trimethylxanthine	coffeine, theine, mateine,	Synonyms	
C <sub>8</sub> H <sub>10</sub> N <sub>4</sub> O <sub>2</sub>		Empirical formula	
3,7-dihydro-1,3,7-trimethyl-1H-purine-2,6-dione		IUPAC Name	
58-08-2		CAS Registry Number	
T56 BN DN FNVNVJ B1 F1 H1		WLN Notation	
CN1C=NC2=C1C(=O)N(C(=O)N2C)C		SMILES	
1S/C8H10N4O2/c1-10-4-9-6-		Inchl	
5(10)7(13)12(3)8(14)11(6)2/h4H,1-3H3		Markush Structure	

	1	2	3	4	5	6	7	8	9	10	11
1	0	1	0	0	0	0	0	0	0	2	0
2	1	0	2	0	0	0	0	0	0	0	0
3	0	2	0	1	0	0	0	1	0	0	0
4	0	0	1	0	1	0	0	0	0	0	0
5	0	0	0	1	0	2	1	0	0	0	0
6	0	0	0	0	2	0	0	0	0	0	0
7	0	0	0	0	1	0	0	0	0	0	0
8	0	0	1	0	0	0	0	0	2	0	0
9	0	0	0	0	0	0	0	2	0	1	0
10	2	0	0	0	0	0	0	0	1	0	1
11	0	0	0	0	0	0	0	0	0	1	0

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Fragment Code  
Fingerprint  
Hash Code

# 化学信息学资源

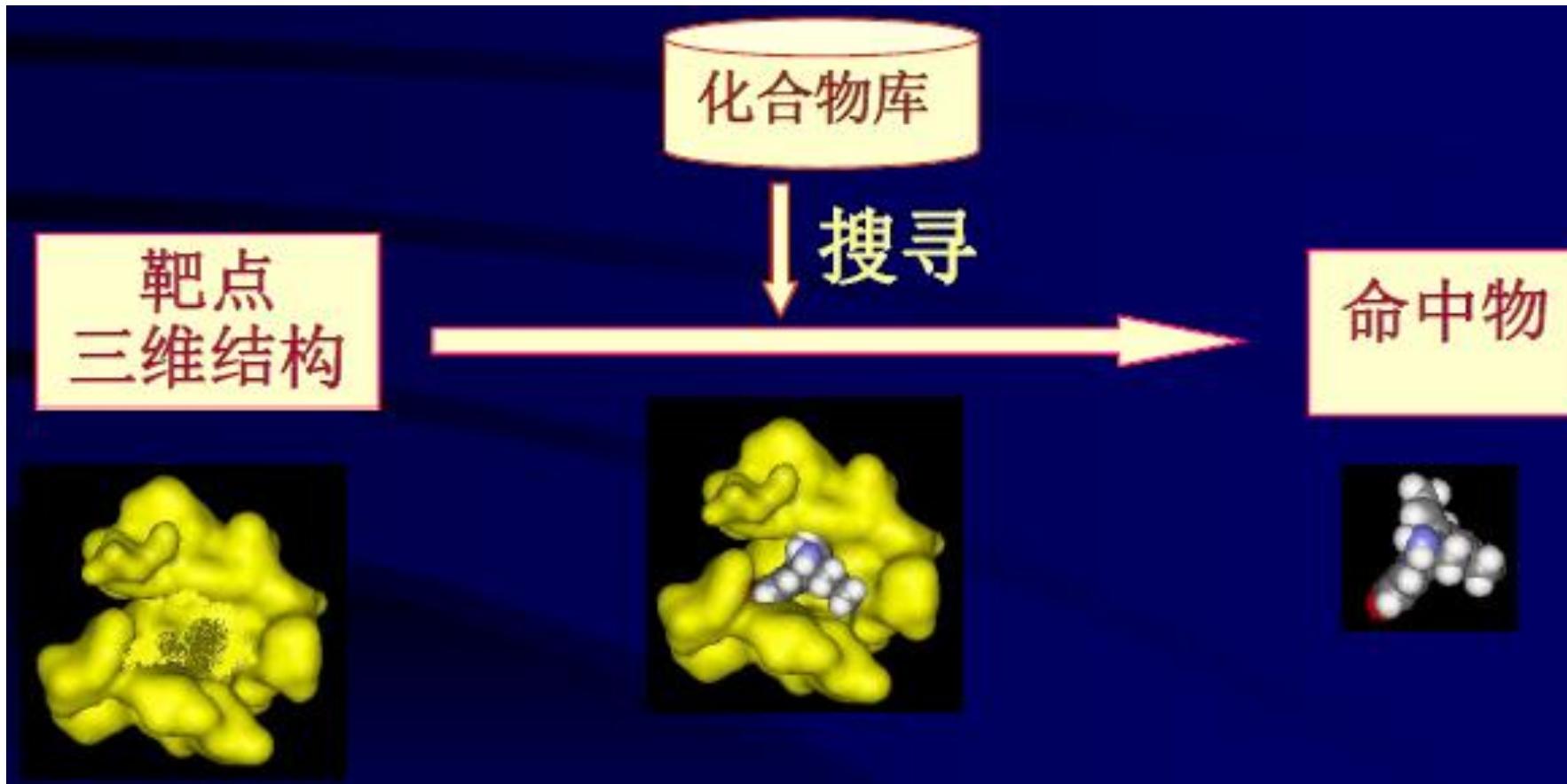


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- FCD (Fine Chemicals Directory) —— MDL 维护。收载约90 000个化合物和20 000种化合物数据，包括化学系统名、俗称、分子式、分子量、供应商、价格、CAS登录号、纯度等。可通过结构式或其它任何数据检索
- ACD (Available Chemicals Directory) ——MDL维护。FCD数据库加上可大批量供货的化学品信息。目前有25万个化合物
- CSD (Cambridge Structure Database) —— 20多万个结晶的3D结构实验数据及相关数据

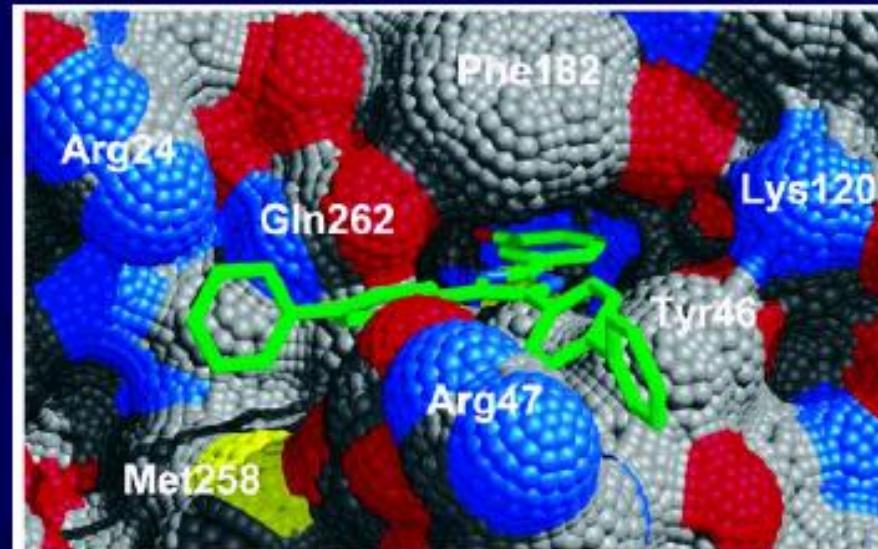
## □ 基于靶点结构的虚拟筛选——对接

■ target-based virtual screening —— docking



## 口例：蛋白酪氨酸磷酸酯酶1B (PTP1B) 抑制剂的发现

经虚拟筛选，再作生物学测试，虚拟筛选的命中率比随机的高通量筛选提高1,700倍



technique	compds tested	hits with IC <sub>50</sub> < 100 $\mu$ M	hits with IC <sub>50</sub> < 10 $\mu$ M	hit rate (%)
HTS	400 000	85	6	0.021
docking	365	127	21	34.8



**ICM<sup>[3]</sup>**

 **AutoDock**  
**AutoDock<sup>[2]</sup>**

  
**GOLD<sup>[2]</sup>**



**Dock<sup>[1]</sup>**

**SCHRÖDINGER**  
**GLIDE<sup>[4]</sup>**

 **Tripos**

**Surflex<sup>[5]</sup>**

  
**accelrys<sup>®</sup>**

**LigandFit<sup>[6]</sup>**

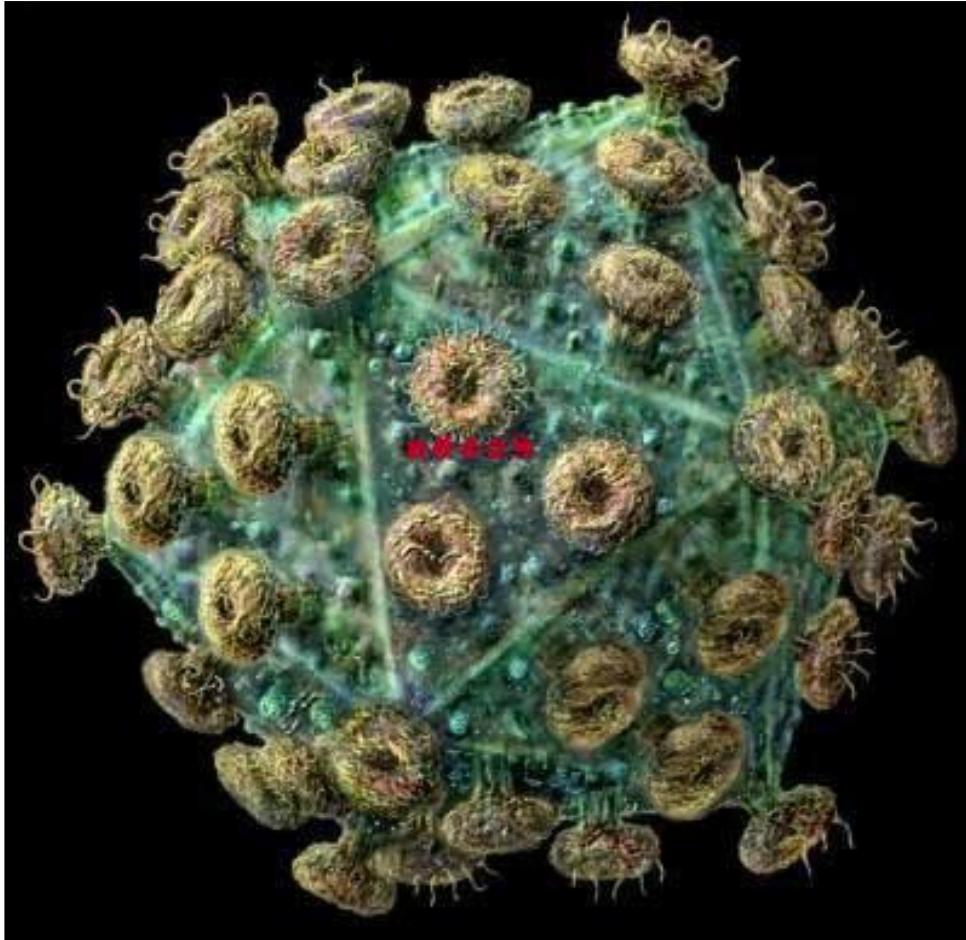
  
**CHEMICAL  
COMPUTING  
GROUP**  
Scalable Software. Scalable Science.

**FlexX<sup>[1]</sup>**

**Conformational  
search engine**

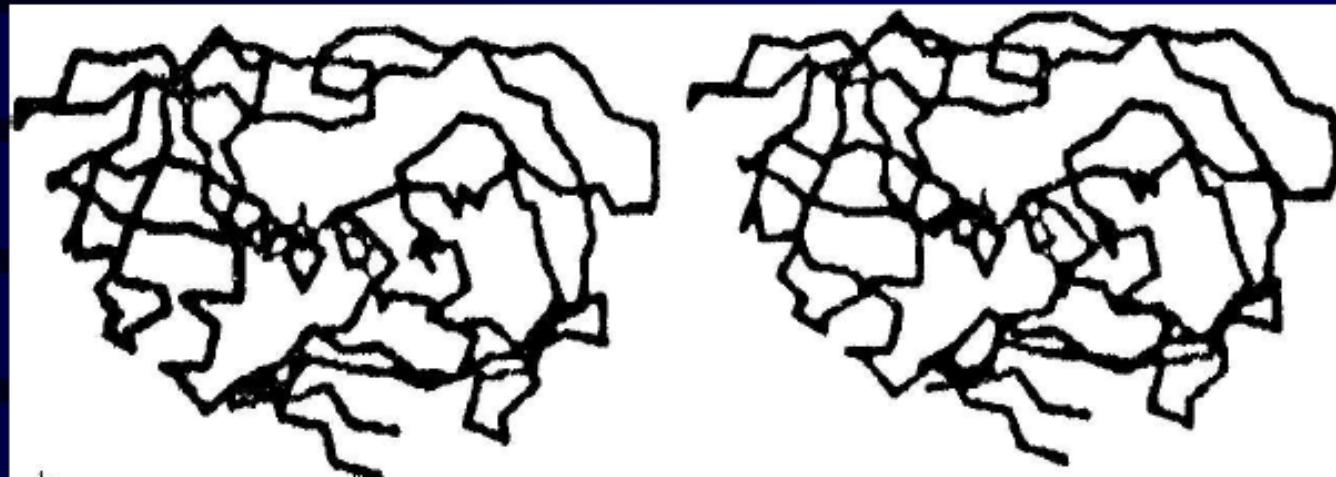
- <sup>1</sup>Incremental construction
- <sup>2</sup>Genetic algorithm
- <sup>3</sup>Pseudobrownian position move
- <sup>4</sup>Systematic search
- <sup>5</sup>Similarity-based search
- <sup>6</sup>Monte Carlo simulation
- <sup>7</sup>Tabu search

## □ [例] 抗艾滋病药物的发现——虚拟筛选



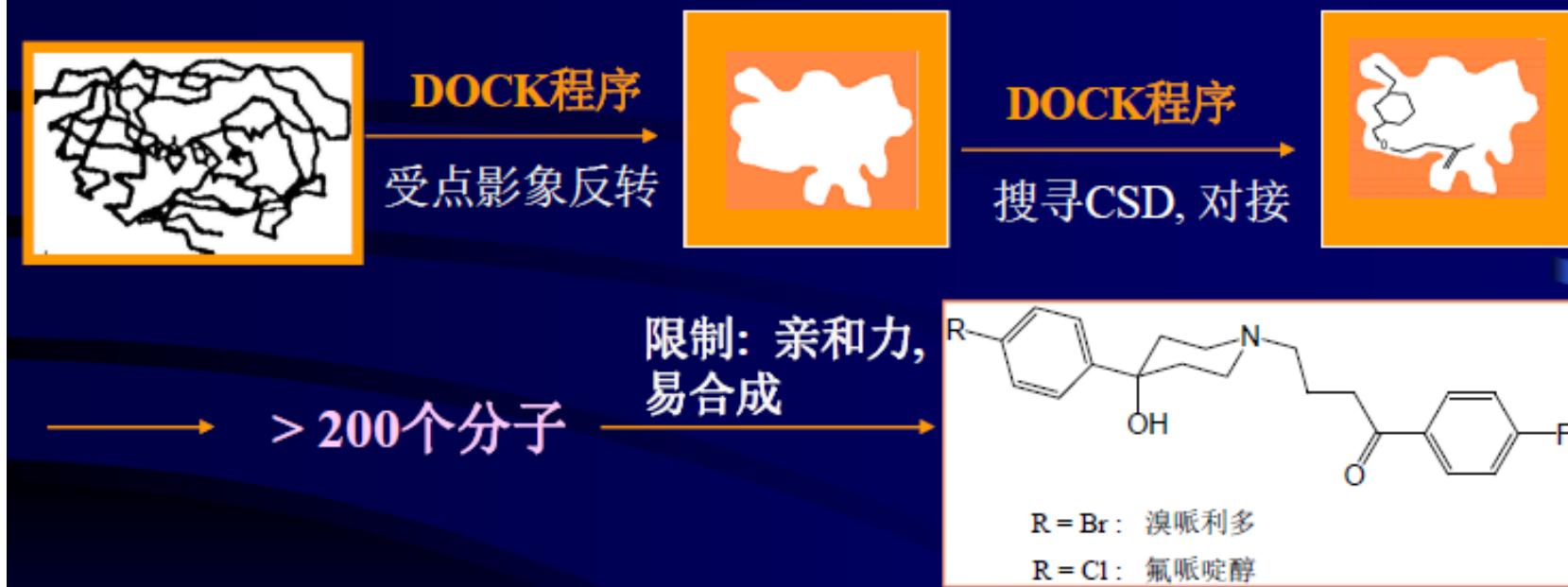
艾滋病病毒, 人类免疫缺陷病毒  
human immunodeficiency virus,  
HIV

- HIV-1蛋白酶（HIV-PR），所催化的水解反应在艾滋病病毒导入人体细胞过程中起着重要的作用



- 高效的HIV-PR抑制剂为治疗艾滋病的有效药物
- 肽类HIV-PR抑制剂生物性质不稳定，吸收性差，易被代谢分解，因此口服给药无效

## 1. X-射线晶体结构 2. 搜寻数据库



## 3. 生物测试: 高选择性, 高活性( $K_i = 0.1 \text{ nM}$ )

- [例] 抗SARS冠状病毒药物的设计
- ——基于SARS-CoV 3CL蛋白酶的虚拟筛选



严重急性呼吸道综合征  
Severe Acute Respiratory Syndrome  
**SARS**



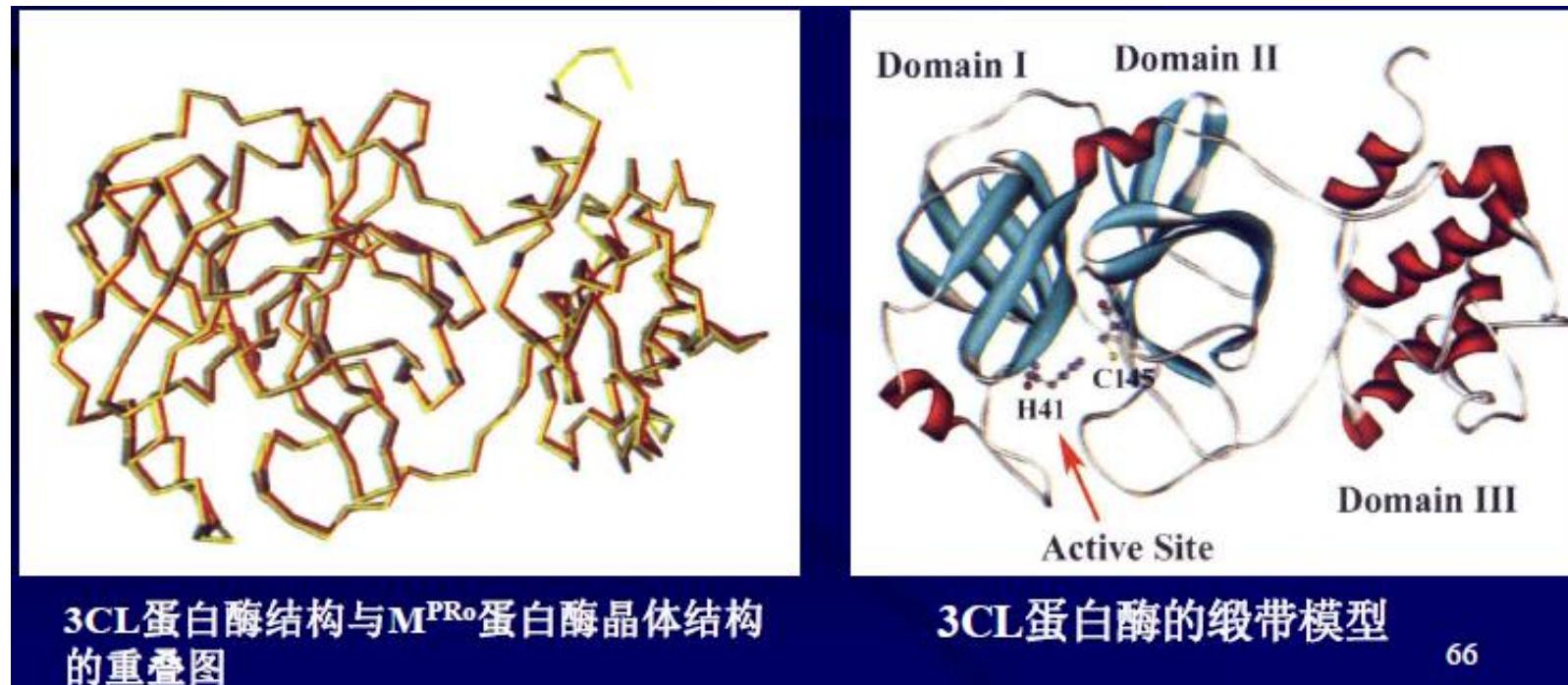
病原体——SARS冠状病毒  
**SARS-CoV**

## □ 步骤1. 同源模建

- (1) 3CL蛋白酶序列(GenBank)与各类冠状病毒蛋白酶序列 (PDB) 作序列分析和相似性分析 (BLAST程序)
  - 人冠状病毒；鼠科肝炎病毒；猪传染性腹泻病毒；猫传染性腹膜炎病毒；禽传染性支气管炎病毒；猪冠状病毒；传染性胃肠炎病毒
- (2) 传染性胃肠炎病毒 (TGEV) 的MPRo与3CL蛋白酶有极高的相似性，特别在底物结合口袋 (活性部位)
- (3) 以TGEV MPro的X-射线晶体结构为模板，模建3CL蛋白酶三维结构 (Sybyl 6.8 / SiteID程序)

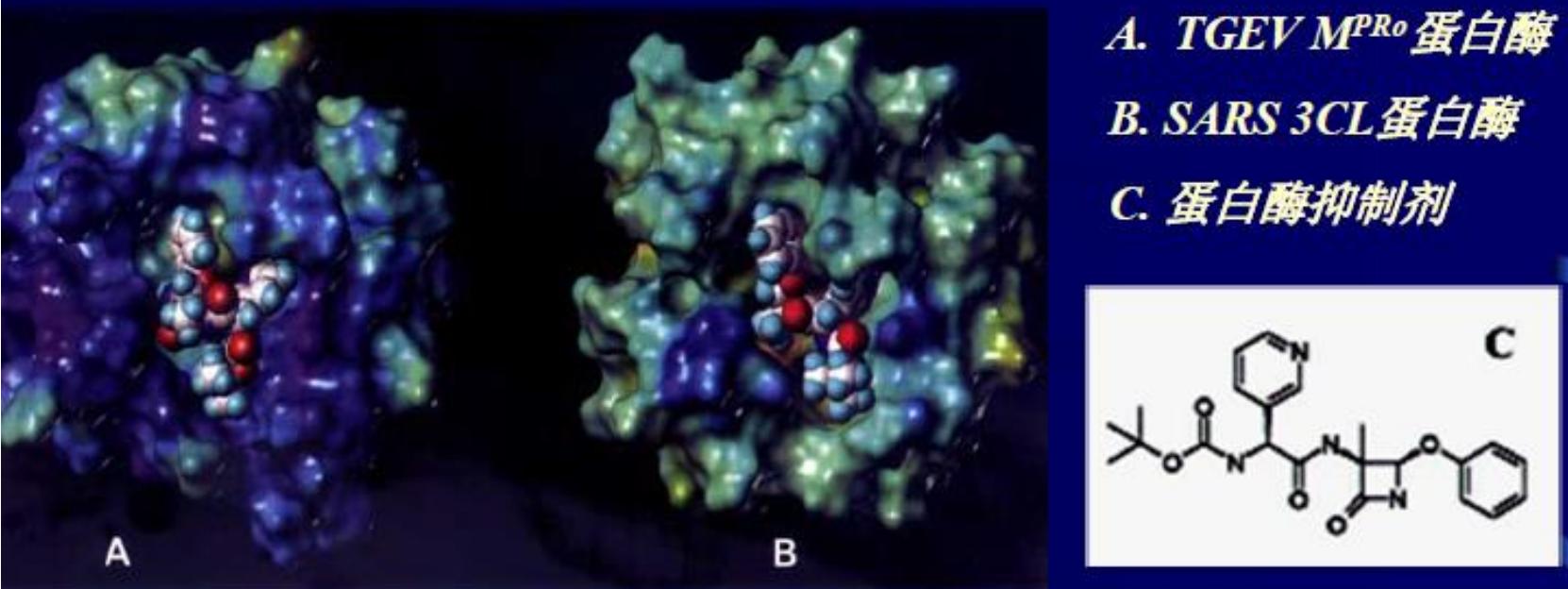
## □ 结果：

- (1) 所建模型与TGEV M<sup>PRo</sup>晶体结构基本重叠
- (2) 3CL蛋白酶的折叠方式与TGEV M<sup>PRo</sup>相同，结合口袋的结构以及空间特征几乎一样



## □ 步骤2. 分析酶-配体作用模型

两种蛋白酶的结合部位 (Sybyl 6.8 / MOLCAD程序) 中，小分子C能以同样的方式与两种酶的结合口袋契合



∴ 3CL蛋白酶模建模型或TGEV M<sup>PRo</sup>的晶体结构均可作为筛选抗SARS药物的结构模型

## □ 步骤3. 虚拟筛选

以SARS冠状病毒3CL蛋白酶三维结构模型和TGEV M<sup>PRO</sup>为筛选模型作虚拟筛选（SGI Origin 3800超级计算机和392CPU的神威1号超级计算机）

ACD数据库、MDDR数据库、SPECS数据库、中国天然产物数据库（CNPD）和国家药物筛选中心内部样品库——共数十万个化合物

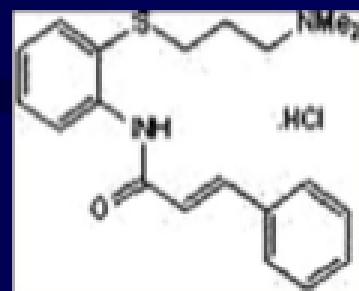
- (1) DOCK 4.0作初筛，选出得分高的前1000个化合物；
- (2) 用Cscore软件和AutoDock 3.0软件作评价，从每个数据库中挑选出100个得分最高的化合物

结果：找到300个可能具有抗SARS冠状病毒潜力的候选化合物

## □ 步骤4. 药理测试

(1) 用病毒3CL蛋白酶分子水平筛选模型筛选候选化合物  
——发现了7个具有高活性的化合物

(2) 在P3实验室中作SARS病毒感染细胞水平的测试，发现5-HT受体拮抗剂（肉桂硫胺，Cinanserin）有明显的抗SARS病毒感染和保护细胞的作用



(3) 申请专利，以CADD作结构优化